MICHIGAN STATE UNIVERSITY
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Analysis and Approximation of a Fractional Cahn-Hilliard Equation

Abstract

We derive a Fractional Cahn-Hilliard Equation (FCHE) by considering a gradient flow in the negative order Sobolev space H^{α} where the choice $\alpha = 1$ corresponds to the classical Cahn-Hilliard equation whilst the choice $\alpha = 0$ recovers the Allen-Cahn equation. It is shown that the equation preserves mass for all positive values of fractional order and that it indeed reduces the free energy. The well-posedness of the problem is established in the sense that the H^1-norm of the solution remains uniformly bounded. We then turn to the delicate question of the boundedness of the solution and establish a pointwise bound for the FCHE in the case where the non-linearity is a quartic polynomial. As a consequence of the estimates, we are able to show that the Fourier-Galerkin method delivers a spectral rate of convergence for the FCHE in the case of a semi-discrete approximation scheme. Finally, we present results obtained using computational simulation of the FCHE for a variety of choices of fractional order. It is observed that the nature of the solution of the FCHE with a general $\alpha > 0$ is qualitatively (and quantitatively) closer to the behaviour of the classical Cahn-Hilliard equation than to the Allen-Cahn equation, regardless of how close to zero be the value of α. An examination of the coarsening rates of the FCHE reveals that the asymptotic rate is rather insensitive to the value of α and, as a consequence, is close to the well-established rate observed for the classical Cahn-Hilliard equation.

This is joint work with Zhiping Mao, Brown University.