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Fractional Derivatives On Cosmic Scales

Abstract

The transport of cosmic rays (CRs, energetic, subatomic particles arriving from outer space) in the interstellar medium is a crucial and open problem in astrophysics. Most modern calculations of galactic CRs propagation (GalProp, Dragon, Usine and other codes) are based on the use of the local diffusion model acceptable for the Brownian motion (Bm) which is unable to adequately describe the diffusion in turbulent media [1-3]. Among a few mathematical models of the turbulent diffusion, the nonlocal model seems to be the most promising, especially in its fractional version. Two reasons validate this statement: 1) the property of self-similarity inherent in both turbulent phenomena and fractional operators, and 2) the fractional corresponding principle conjugating fractional-order differential equations for turbulent media with integer-order differential equations for laminar media. Such model was constructed by Lagutin, Nikulin, Uchaikin in 2000 [4], and up to 2010 have been used for calculations of energy spectra, anisotropy coefficients, mass composition and other observable characteristics of CRs. Comparison of the results with Monte Carlo and observation data showed agreement better than the standard diffusion model except the cases linked with time variable. The reason of that was the uncertainty in the speed of the LNU-model. Involving the relativistic speed limit [5] gave a new breath to the nonlocal model turned its basic equation to an equation with the material derivative of a fractional order. The modified in such a way model was described in [5,6] and called NoRD (Nonlocal Relativistic Diffusion) model. In this report, we discuss the present state, unsolved problems and prospects of the fractional approach to description of CR propagation.

Unlike the usual material derivative whose form is independent of boundaries and boundary conditions, the fractional analogue depends on them and, it is necessary to specify not only the properties of the sought function at the domain boundaries, but also its values outside the domain. The problem of initialization is discussed. The fact that the NoRD-propagators are characterized by the presence of fronts near the ballistic restriction provides interesting phenomena such as anisotropy inversion in nonhomogeneous medium. It can help to solve the CR anisotropy problem in the TeV-range.

From specified spectra of magnetic turbulence we obtained the equation containing tempered fractional operator [7] describing tempered Levy walk. Truncation factor is energy dependent and due to wide distribution of CR energy we observe different transport regimes for particle ensemble at chosen space scale. The tempered fractional concept allows us to coordinate the NoRD-model with the diffusion approach at small energies. In frames of the NoRD-model, we consider the interpretation of `knee' in the energy spectrum, estimate anisotropy and mass composition of galactic CRs for single source and for supernova ensemble. We hope that interpretation of the collected experimental data on CRs within the fractional approach can help to solve the mystery of cosmic rays origin.

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