MICHIGAN STATE UNIVERSITY
Department of Statistics and Probability

A Workshop on Future Directions in Fractional Calculus Research and Applications

Richard L. Magin
University of Illinois at Chicago

A fractional derivative model of anomalous diffusion in white and gray matter

Abstract

Non-Gaussian (anomalous) diffusion is wide spread in biological tissues where its effects modulate chemical reactions and membrane transport. When viewed using magnetic resonance imaging (MRI), anomalous diffusion in the white and gray matter of the brain is characterized by a persistent or 'long tail' behavior in the decay of the diffusion signal. Recent MRI studies have used the fractional derivative to describe such diffusion dynamics. In this study, we modify the Bloch-Torrey equation by introducing fractional order time and space derivatives into Fick's second law of diffusion. In this model the fraction derivative order is directly connected to the jump increments and waiting times of the diffusion trajectory (within the context of the continuous time random walk model of Brownian motion). Furthermore, in addition to the order of the fractional derivatives, the spectral entropy is used to measure of the overall complexity of diffusion. As an example, we apply this new model for the characterization of diffusion in fixed samples of the rat and mouse brains, as well as to human MRI studies of brain tumors.