MICHIGAN STATE UNIVERSITY
Department of Statistics and Probability

A Workshop on Future Directions in Fractional Calculus Research and Applications

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Promoting the Use of Fractional Calculus in the Modeling Engineering Systems

Abstract

In the mathematics and physics literature it is well known that the presence of heterogeneity in a transport system can lead to anomalous behaviors and that such behaviors can be described by introducing fractional derivative terms into the classic transient-advection-diffusion model. An important area where the use and development of fractional calculus techniques has enjoyed a widespread and meaningful application is in the modeling of transport in natural porous media.

Outside of this domain, however, apart from some preliminary forays with "toy" models, it is fair to say that the engineering use of fractional calculus transport models has been limited. The premise of this presentation is that the advancement and motivation of the wider use fractional calculus in modeling engineering systems requires:

- Demonstrating how realistic engineered systems can exhibit unambiguous signals of anomalous behavior.
- Deriving sound physically based fractional calculus model that can describe these behaviors.
- Developing computational algorithms that can resolve these models

Here we illustrate these key items by considering the classical heat transfer problem of heat conduction phase change (the Stefan problem). First we will demonstrate, through direct simulation of melting in a medium containing heterogeneity, how the progress of the phase change can exhibit anomalous signals.

Then we will discuss some physical issues related to the building of a factional calculus model of this process. We will conclude by outlining how numerical solution strategies for these fractional calculus models, on unstructured finite element meshes, might be constructed.