A Workshop on Future Directions in Fractional Calculus Research and Applications

Department of Statistics and Probability
Michigan State University

Zhi Zhou

Numerical Analysis for Time-Fractional Evolution Equations

Abstract

The time-fractional evolution model has received much attention in recent years, since it can adequately capture the dynamics of anomalous diffusion processes. Due to the nonlocality of the fractional derivative, high-order schemes are of immense interest. However, in comparison with its standard parabolic counterpart, the major technical challenge in the development and analysis of robust numerical schemes lies in the limited smoothing property of the solution operator.

In this talk, I will start with spatial semidiscrete scheme by continuous piecewise linear Galerkin finite element approximation. We establish almost optimal error estimates (with respect to the data regularity), including the cases of smooth and nonsmooth data. Then I will turn to the time stepping schemes and present some simple examples to show how the insufficient regularity significantly deteriorates the convergence rate of time stepping schemes. We consider the convolution quadrature generated by linear multistep methods, and present an initial-correction strategy to restore the optimal order for any fixed $t > 0$. This strategy can be applied to reinstate the optimal order of various methods, such as $L1$ scheme, Crank-Nicolson scheme and convolution quadrature by high order BDFs. Our argument will be focused on the fractional diffusion equation ($0 < \alpha < 1$), and then extended to the diffusion-wave equation ($1 < \alpha < 2$).