Goodness-of-fit tests for long memory moving average marginal density[1]

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Abstract

This paper addresses the problem of fitting a known density to the marginal error density of a stationary long memory moving average process when its mean is known and unknown. In the case of unknown mean, when mean is estimated by the sample mean, the first order difference between the residual empirical and null distribution functions is known to be asymptotically degenerate at zero, and hence can not be used to fit a distribution up to an unknown mean. In this paper we show that by using a suitable class of estimators of the mean, this first order degeneracy does not occur. We also investigate the large sample behavior of tests based on an integrated square difference between kernel type error density estimators and the expected value of the error density estimator based on errors. The asymptotic null distributions of suitably standardized test statistics are shown to be chi-square with one degree of freedom in both cases of the known and unknown mean. In addition, we discuss the consistency and asymptotic power against local alternatives of the density estimator based test in the case of known mean. A finite sample simulation study of the test based on residual empirical process is also included.

1 Introduction

The problem of fitting a parametric family of distributions to a probability distribution, known as the goodness-of-fit testing problem, is classical in statistics, and well studied when the underlying observations are i.i.d. See, for example, Durbin (1973, 1975), Khmaladze (1979, 1981), D'Agostino and Stephens (1986), among others.

A discrete time stationary stochastic process with finite variance is said to have long memory if its autocorrelations tend to zero hyperbolically in the lag parameter, as the lag tends to infinity, but their sum diverges. The importance of these processes in econometrics, hydrology and other physical sciences is abundantly demonstrated in the works of Beran

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