We consider the classical problem of testing for the mean vector of a multivariate distribution when the covariance matrix is unknown and the companion problem of testing the equality of means of two populations when the common covariance matrix is unknown. A well-known test procedure is the Hotelling's $T^2$ test. But it is not well-defined when the dimension is larger than the sample size. We consider a test based on a regularized version of the Hotelling's $T^2$ statistic through a modification of the sample covariance matrix similar to what is used in ridge regression. We study the properties of the proposed test procedures under the scenario when the dimension and sample sizes are comparable and compare them with various other tests proposed in the literature.