## Example 1.1

```
Graph > Stem-and-Leaf
Stem-and-leaf of fundrais N = 60
Leaf Unit = 1.0
19 0 0111112222333333344
(17) 0 55556666666778888
24 1 0001222244
14 1 55666789
6 2 01
4 2 6
3 3 4
2
4}
5
1 6
1 6
1 7
7
```

Fundraising expenses (\% of total expenses) for organized charities have a unimodal distribution, skewed to the right ( + ) with two unusual observations at 48 and 83 percent.

Graph > Histogram


## Example 1.16

Stat > Basic Statistics > Display Descriptive Statistics (select C1 \% Copper)
Results for: exp01-16.mtw
Descriptive Statistics: \% Copper

| Variable | $N$ | $N^{*}$ | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| \% Copper | 26 | 0 | 3.654 | 0.303 | 1.547 | 2.000 | 2.700 | 3.350 | 3.875 | 10.100 |

Conclusion: The average (or mean) copper content is 3.65 \%. Half of the Bidri samples had copper content of less than $3.35 \%$ and half had a greater copper content.

## Example 1.18 (p. 41)

Stat > Basic Statistics > Display Descriptive Statistics (select C1)
Results for: exp1-18.mtw
Descriptive Statistics: C1

| Variable | N | $N^{*}$ | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| C1 | 19 | 0 | 86.32 | 5.35 | 23.32 | 40.00 | 70.00 | 90.00 | 98.00 | 125.00 |

Conclusion: A typical pit depth is 86 thousandths of an inch, with a typical spread of 23 thousandths of an inch either way.

Graph > Boxplot (Simple, select C1)


Conclusion: The distribution of pit depths is positively skewed with no apparent outliers.

## Example 3.32

Calc $>$ Probability Distributions $>$ Binomial ( $n=15, p=0.2, C D F, x=8$ )
Cumulative Distribution Function
Binomial with $\mathrm{n}=15$ and $\mathrm{p}=0.2$
$x \quad P(x<=x)$
80.999215

Conclusion: Probability that at most 8 fail the test is .9992
Calc $>$ Probability Distributions $>$ Binomial ( $n=15, p=0.2, p m f, x=8$ )

## Probability Density Function

Binomial with $\mathrm{n}=15$ and $\mathrm{p}=0.2$
$x \quad P(X=x)$
80.0034548

Conclusion: Probability that exactly 8 fail the test is .003

Calc $>$ Probability Distributions $>$ Binomial $(n=15, p=0.2$, inverse pmf, prob=.5)

Inverse Cumulative Distribution Function
Binomial with $\mathrm{n}=15$ and $\mathrm{p}=0.2$

```
x P( X <= x ) X P( X <= x )
2 0.398023 3 0.648162
```

Conclusion: Median is 3 . There is at least a $50 \%$ chance that $<=3$ fail the test, and there is at least a $50 \%$ chance that $>=3$ fail the test.

## Example 3.40

Calc > Probability Distributions > Poisson (Probability, mean=2, input constant 1)

## Probability Density Function

```
Poisson with mean = 2
x P( X = x )
10.270671
```

Conclusion: Probability that exactly one error is found would be $27 \%$
Calc > Probability Distributions > Poisson (Cumulative probability, mean=2, input constant 3)

## Cumulative Distribution Function

```
Poisson with mean = 2
x P( X <= x )
3 0.857123
```

Conclusion: Probability that at most three errors are found would be $85.7 \%$

Calc > Probability Distributions > Poisson (Inverse cumulative probability, mean=2, input constant 0.75)

## Inverse Cumulative Distribution Function

```
Poisson with mean = 2
x P( X <= x ) X P( X <= x )
2 0.676676 3 0.857123
```

Conclusion: Third quantile Q3 is equal to 3 . There is at least a $75 \%$ chance that <=3 errors are found, and there is at least a $25 \%$ chance that $>=3$ errors are found.

## Example 4.16

Calc > Probability Distributions > Normal (Cumulative probability, mean 1.25, standard deviation 0.46 , input constant 1.00)

## Cumulative Distribution Function

```
Normal with mean = 1.25 and standard deviation = 0.46
```

$x \quad P(X<=x)$
10.293400

Calc > Probability Distributions > Normal (Cumulative probability, mean 1.25, standard deviation 0.46 , input constant 1.75)

## Cumulative Distribution Function

Normal with mean $=1.25$ and standard deviation $=0.46$

```
    P( X <= x )
1.75 0.861472
```

Conclusion: Then $\mathrm{P}(1.00<\mathrm{X}<1.75)=.861-.293=.568$ so the probability that reaction time is between 1.00 and 1.75 seconds is $56.8 \%$

Calc > Probability Distributions > Normal (Cumulative probability, mean 1.25, standard deviation 0.46, input constant 2.00)

## Cumulative Distribution Function

Normal with mean $=1.25$ and standard deviation $=0.46$

```
X P( X <= X )
```

20.948495

Conclusion: Then $\mathrm{P}(\mathrm{X}>2)=1-.948=.052$ so the probability that reaction time exceeds 2.0 seconds is $5.2 \%$

Calc > Probability Distributions > Normal (Inverse cumulative probability, mean 1.25 , standard deviation 0.46 , input constant 0.99 )

## Inverse Cumulative Distribution Function

Normal with mean $=1.25$ and standard deviation $=0.46$
$\begin{array}{rrr}P(X<= & X \\ 0.99 & 2.32012\end{array}$

Conclusion: $99^{\text {th }}$ percentile is 2.32 . There is a $99 \%$ chance that reaction time is less than 2.32 seconds

## Example 4.24

Calc > Probability Distributions > Gamma (Cumulative probability, shape=8, scale $=15$, input constant 60)

## Cumulative Distribution Function

Gamma with shape $=8$ and scale $=15$

```
x P( X <= x )
```

Calc > Probability Distributions > Gamma (Cumulative probability, shape=8, scale $=15$, input constant 120)

## Cumulative Distribution Function

Gamma with shape $=8$ and scale $=15$

```
    x P( X <= x )
120 0.547039
```

Conclusion: Then $\mathrm{P}(60<\mathrm{X}<120)=.547-.051=.496$ so the probability that a mouse survives between 60 and 120 weeks is $49.6 \%$

Calc > Probability Distributions > Gamma (Cumulative probability, shape=8, scale $=15$, input constant 30 )

## Cumulative Distribution Function

```
Gamma with shape = 8 and scale = 15
```

```
X P( X < X X )
```

Conclusion: Then $\mathrm{P}(\mathrm{X}>30)=1-.001=.999$ so the probability that a mouse survives at least 30 weeks is $99.9 \%$

Calc > Probability Distributions > Gamma (Inverse cumulative probability, shape $=8$, scale $=15$, input constant 0.25 )

## Inverse Cumulative Distribution Function

Gamma with shape $=8$ and scale $=15$


Conclusion: First quantile Q1 is equal to 89.3. There is a $25 \%$ chance that a mouse will survive less that 89.3 weeks.

## Example 4.25

Calc > Probability Distributions > Weibull (Cumulative probability, shape=2, scale=10, input constant 10)

## Cumulative Distribution Function

Weibull with shape $=2$ and scale $=10$

```
x P( X <= x )
10 0.632121
```

Conclusion: There is a $63 \%$ chance that nitrous oxide emissions are less than 10 .
Calc > Probability Distributions > Weibull (Cumulative probability, shape=2, scale=10, input constant 25)

Cumulative Distribution Function

```
Weibull with shape = 2 and scale = 10
    x P( X <= x )
25 0.998070
```

Conclusion: There is a $99.8 \%$ chance that nitrous oxide emissions are less than 25.
Calc > Probability Distributions > Weibull (Inverse cumulative probability, shape=2, scale=10, input constant 0.95)

Inverse Cumulative Distribution Function

```
Weibull with shape = 2 and scale = 10
P( X <= x ) x
    0.95 17.3082
```

Conclusion: $95 \%$ of NOx emissions are less than 17.3.

## Example 4.30

## Graph > Probability Plot (Single, select data)

## Results for: $\exp 4-30 . \mathrm{mtw}$

## Probability Plot of $\mathrm{X}(\mathbf{1})$ :



Conclusion: The distribution of dialectric breakdown voltage data appears to fit a normal distribution with mean 27.79 and standard deviation 1.462.

## Example 7.11

## Stat > Basic Statistics > One sample-t (select column C1)

## Results for: EXP07-11.MTW

One-Sample T: rupture

| Variable | N | Mean | StDev | SE Mean | $95 \% \mathrm{CI}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| rupture | 30 | 7203.2 | 543.5 | 99.2 | $(7000.2, ~ 7406.2)$ |

Conclusion: $95 \%$ sure that average strength is between 7000 and 7406 psi.
Graph > Probability Plot (Single, select data)


Conclusion: OK, data appears normal, t-test procedure is valid.

## Example 8.9

Stat > Basic Statistics > One sample-t > Options (Alternative: not equal)
One-Sample T: conc

```
Test of mu = 4 vs not = 4
```

| Variable | $N$ | Mean | StDev | SE Mean | $95 \%$ CI | $T$ | $P$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| conc | 5 | 3.814 | 0.718 | 0.321 | $(2.922,4.706)$ | -0.58 | 0.594 |

Calculation: Reject $\mathrm{H}_{0}$ at the $95 \%$ level if $\mathrm{T}>\mathrm{t}_{.025,4}=2.776$. Since $\mathrm{T}=-0.58$, do not reject. [Or, do not reject because the P -value is $\mathrm{P}=0.594>.05$.]

Conclusion: Insufficient evidence to be $95 \%$ sure that average concentration differs from $4 \mathrm{mg} / \mathrm{mL}$.

Graph > Probability Plot (Single, select data)


Conclusion: OK, data appears normal, t-test procedure is valid.

## Example 8.11

Stat > Basic Statistics > 1 Proportion > (Number of events: 16, Number of trials: 91, Perform hypothesis test, Hypothesized proportion: 0.15, Options: Confidence level: 90, Alternative: greater than, Use test and interval based on normal distribution)

## Test and Cl for One Proportion

Test of $p=0.15$ vs $p>0.15$

|  |  |  | $90 \%$ Lower |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Sample | X | N | Sample p | Bound | Z-Value | P-Value |  |
| 1 | 16 | 91 | 0.175824 | 0.124684 | 0.69 | 0.245 |  |

Using the normal approximation.
Since the test statistic $\mathrm{z}=0.69$ is less than the $90 \%$ cut off value of 1.282 , we do not reject the null hypothesis.

Conclusion: Insufficient evidence to be $90 \%$ sure that more than $15 \%$ of corks are bad.

## Example 8.18

Stat > Basic Statistics > One sample-t > Options (Alternative: not equal)
One-Sample T: conc

```
Test of mu = 4 vs not = 4
```

| Variable | N | Mean | StDev | SE Mean | $95 \%$ CI | T | P |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| conc | 5 | 3.814 | 0.718 | 0.321 | $(2.922,4.706)$ | -0.58 | 0.594 |

Conclusion: We are $41.6 \%$ sure that average concentration differs from $4 \mathrm{mg} / \mathrm{mL}$. [There is insufficient evidence to conclude that average concentration differs from 4 $\mathrm{mg} / \mathrm{mL}$.]

Graph > Probability Plot (Single, select data)


Conclusion: OK, data appears normal, t-test procedure is valid.

## Example 9.9

Stat > Basic Statistics > One sample-t > Options (level, alternative)
One-Sample T: Differen
Test of mu $=0$ vs not $=0$

| Variable | $N$ | Mean | StDev | SE Mean | $95 \%$ CI | T | $P$ |
| :--- | ---: | ---: | ---: | ---: | :---: | ---: | ---: |
| Differen | 16 | 6.75000 | 8.23408 | 2.05852 | $(2.36237,11.13763)$ | 3.28 | 0.005 |

Conclusion: We are 99.5\% sure that the average proportion of time at which arm angle is less than 30 degrees has changed after work conditions were changed (since $\mathrm{p}=.005$ ).
Also, we are $95 \%$ sure that the average proportion of time at which arm angle is less than 30 degrees, after the change in working conditions, is between $2 \%$ and $11 \%$ less than it was before (based on the 95\% CI).

Graph > Probability Plot (Single, select data)


Conclusion: OK, data appears normal, t-test procedure is valid.

Alternative procedure based on two-sample t-test

## Stat > Basic Statistics > 2 sample-t > Options (level, alternative)

## Two-Sample T-Test and CI: Before:, After:

Two-sample T for Before: vs After:


Conclusion: We are $92.4 \%$ sure that the average proportion of time at which arm angle is less than 30 degrees has changed after work conditions were changed (since $p=.076$ ). Also, we are $95 \%$ sure that the average amount of this change is between $-0.7 \%$ and $+14.3 \%$ (based on the $95 \% \mathrm{CI}$ ). Note that the paired t-test gives more definitive results!

Graph > Probability Plot (Single, select data)


Conclusion: OK, each data set appears normal, 2-sample t-test procedure is valid.

## Example 9.11

Stat > Basic Statistics > 2 Proportions (Options: Alternative: greater than, Use pooled estimate for $p$ test)

## Test and CI for Two Proportions

```
\begin{tabular}{lrrr} 
Sample & X & \(N\) & Sample p \\
1 & 81 & 549 & 0.147541 \\
2 & 141 & 730 & 0.193151
\end{tabular}
Difference = p (1) - p (2)
Estimate for difference: -0.0456097
95% upper bound for difference: -0.0110060
Test for difference = 0 (vs < 0): Z = -2.13 P-Value = 0.017
Fisher's exact test: P-Value = 0.019
```

Since p-value is 0.017 we reject the null hypothesis at level 0.01 , but not at level 0.01 .
Conclusion: We are $98.3 \%$ sure that aspirin use increases the 15 year survival rate for colorectal cancer victims.

## Examples 12.1 and 12.2

Graph > Scatterplot


Conclusion: There appears to be a strong positive linear relation between $y=$ OSA and $\mathrm{x}=$ palprebal fissure width.


Conclusion: There appears to be a weak negative linear relation between $\mathrm{y}=$ mean crown dieback and $\mathrm{x}=\mathrm{pH}$.

## Example 12.4

Stat > Regression > Regression (Storage: Residuals)
Results for: EXP12-04.MTW
Regression Analysis: $y$ versus $x$

| Predictor | Coef | SE Coef | T | P |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Constant 7 | 75.212 | 2.984 | 25.21 | 0.000 |  |
| $x \quad-0$. | -0.20939 | 0.03109 | -6.73 | 0.000 |  |
| $S=2.56450$ | $\mathrm{R}-\mathrm{Sq}=79.1 \%$ |  | $\mathrm{R}-\mathrm{Sq}(\mathrm{adj})=77.3 \%$ |  |  |
| Analysis of Variance |  |  |  |  |  |
| Source | DF | SS | MS | F | P |
| Regression | 1 | 298.25 | 298.25 | 45.35 | 0.000 |
| Residual Error | rror 12 | 78.92 | 6.58 |  |  |
| Total | 13 | 377.17 |  |  |  |

Conclusion: The true mean cetane number y for diesel fuel with iodine value x is estimated to be $y=75.2-0.209 x$, with a typical spread of 2.6. This regression model explains $77.3 \%$ of the variations in cetane number in terms of variations in iodine value.

## Stat > Regression > Fitted Line Plot



Conclusion: The regression line provides a reasonably good fit to the data, indicating that $\mathrm{x}=$ iodine value can give a useful model to predict $\mathrm{y}=$ cetane number.



Conclusion: Probability plot (left) indicates that the residuals fit a normal distribution. Scatterplot (right) indicates the residual seem to be independent and identically distributed. Hence the basic assumptions of the regression model are satisfied.

## Example 12.11

Stat > Regression > Regression (Storage: Residuals)
Results for: exp12-11.mtw
Regression Analysis: y: versus x:

```
The regression equation is
y: = 126 - 0.918 x:
\begin{tabular}{lrrrr} 
Predictor & Coef & SE Coef & T & P \\
Constant & 126.249 & 2.254 & 56.00 & 0.000 \\
x: & -0.9176 & 0.1460 & -6.29 & 0.000
\end{tabular}
S = 2.94100 R-Sq = 75.2% R-Sq(adj) = 73.3%
Analysis of Variance
\begin{tabular}{lrrrrr} 
Source & DF & SS & MS & F & P \\
Regression & 1 & 341.73 & 341.73 & 39.51 & 0.000 \\
Residual Error & 13 & 112.44 & 8.65 & & \\
Total & 14 & 454.17 & & &
\end{tabular}
```

Unusual Observations

| Obs | x: | $y$ : | Fit | SE Fit | Residual | St | Resid |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 10.0 | 124.000 | 117.073 | 1.008 | 6.927 |  | 2.51 R |
| R d | tes | n obse | on wi | a la | standa |  | resid |

Conclusion: The true mean density for mortar with an air content of $\mathrm{x} \%$ is estimated to be $\mu=126-0.918 \mathrm{x} \mathrm{lb} / \mathrm{cu}-\mathrm{ft}$, with a typical spread of $3 \mathrm{lb} / \mathrm{cu}-\mathrm{ft}$. This regression model explains $75.2 \%$ of the variations in density in terms of variations in air content.

The $t$ statistic is -6.29 with a p-value of 0.000 , so we are virtually certain that the regression model provides a useful predictor of mortar density in terms of air content.

## Stat > Regression > Fitted Line Plot



Conclusion: The regression line provides a reasonably good fit to the data, indicating that $\mathrm{x}=$ air content can give a useful model to predict $y=d e n s i t y$.


Conclusion: Probability plot (left) indicates that the residuals fit a normal distribution. Scatterplot (right) indicates the residual seem to be independent and identically distributed. Hence the basic assumptions of the regression model are satisfied. Note: The p-value of 0.275 in the probability plot indicates that there is insufficient evidence to reject the null hypothesis of a normal fit.

## Example 12.13

Stat > Regression > Regression (Storage: Residuals, Options: Prediction intervals for new observation: 45)

## Results for: exp12-13.mtw

Regression Analysis: $y$ : versus $x$ :

```
The regression equation is
y: = 27.2 - 0.298 x:
\begin{tabular}{lrrrr} 
Predictor & Coef & SE Coef & T & P \\
Constant & 27.183 & 1.651 & 16.46 & 0.000 \\
X: & -0.29756 & 0.04116 & -7.23 & 0.000
\end{tabular}
S = 2.86403 R-Sq = 76.6% R-Sq(adj) = 75.1%
Analysis of Variance
\begin{tabular}{lrrrrr} 
Source & DF & SS & MS & F & P \\
Regression & 1 & 428.62 & 428.62 & 52.25 & 0.000 \\
Residual Error & 16 & 131.24 & 8.20 & & \\
Total & 17 & 559.86 & & &
\end{tabular}
Predicted Values for New Observations
New
Obs Fit SE Fit 95% CI 95% PI
    1 13.793 0.758 (12.185, 15.400) (7.512, 20.073)
Values of Predictors for New Observations
New
Obs x:
    145.0
```

Conclusion: For cement samples with a carbonation depth of 45 mm , we are $95 \%$ sure that the average strength is between 12.2 PMa and 15.4 MPa . For one sample of cement with a carbonation depth of 45 mm , we are $95 \%$ sure that the strength of this individual sample is between 7.5 PMa and 20.1 MPa.

The regression model y = 27.2-0.298 x gives the estimated average strength y in MPa for concrete samples with a given carbonation depth $x$ in $m m$. The $t$-value of -7.23 and the corresponding $p$-value of 0.000 indicates strong evidence that there is a positive linear relation between these two variables.

Stat > Regression > Fitted Line Plot


Conclusion: The regression line provides a reasonably good fit to the data, indicating that $\mathrm{x}=$ carbonation depth content can give a useful model to predict $\mathrm{y}=$ strength with a typical error of 2.86 MPa. $77 \%$ of the variations in strength can be attributed to variations in carbonation depth.


Conclusion: Probability plot (left) indicates that the residuals fit a normal distribution. Scatterplot (right) indicates the residuals seem to be independent and identically distributed. Hence the basic assumptions of the regression model are satisfied. Note: The p-value of 0.242 in the probability plot indicates that there is insufficient evidence to reject the null hypothesis of a normal fit.

## Example 12.16

Stat > Regression > Regression (Storage: Residuals)

Regression Analysis: y versus $\mathbf{x}$

```
The regression equation is
y = 1.00 + 93.4 x
\begin{tabular}{lrrrr} 
Predictor & Coef & SE Coef & T & P \\
Constant & 0.998 & 2.703 & 0.37 & 0.717 \\
x & 93.38 & 24.36 & 3.83 & 0.002
\end{tabular}
S = 3.89192 R-Sq = 51.2% R-Sq(adj) = 47.7%
Analysis of Variance
\begin{tabular}{lrrrrr} 
Source & DF & SS & MS & F & P \\
Regression & 1 & 222.48 & 222.48 & 14.69 & 0.002 \\
Residual Error & 14 & 212.06 & 15.15 & & \\
Total & 15 & 434.54 & & &
\end{tabular}
```

Unusual Observations

| Obs | $x$ | $y$ | Fit | SE Fit | Residual | St Resid |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 12 | 0.074 | 16.600 | 7.908 | 1.210 | 8.692 | $2.35 R$ |

$R$ denotes an observation with a large standardized residual.
Stat > Basic Statistics > Correlation
Correlations: $\mathbf{x}, \mathrm{y}$
Pearson correlation of $x$ and $y=0.716$
P -Value $=0.002$

Conclusion: The correlation of . 716 indicates a strong positive relation between ozone and carbon concentrations. The $p=$ value of 0.002 indicates we are $99.8 \%$ sure there is a linear relation between ozone and carbon concentrations. Note that the p-values for the slope and the correlation are identical, this is always the case!

The regression model $\mathrm{y}=1.00+93.4 \mathrm{x}$ gives the estimated average carbon concentration in $\mu \mathrm{g} / \mathrm{mm}^{3}$ for air samples with a given ozone concentration x ppm. Variations in ozone concentration account for $51 \%$ of the variations in carbon concentration.

## Stat > Regression > Fitted Line Plot



Conclusion: The regression line provides a reasonably good fit to the data, indicating that $\mathrm{x}=\mathrm{ozone}$ concentration can give a useful model to predict $\mathrm{y}=$ carbon concentration with a typical error of $3.89 \mu \mathrm{~g} / \mathrm{mm}^{3}$.


Conclusion: Probability plot (left) indicates that the residuals fit a normal distribution. Scatterplot (right) indicates the residuals seem to be independent and identically distributed. Hence the basic assumptions of the regression model are satisfied.

