

# STT200

## Prep for Final Exam

### Wednesday, April 30, 3 - 5 pm, B106 Wells Hall

All needed paper and tables will be provided.  
As usual, no electronics, headphones, notes or books.

**The exam will be comprehensive over:**

#### **Probability**

Rules of Multiplication and Addition  
Total probability  
Conditional probability, Independence, Bayes  
Trees, Venn, Balls from boxes, Dice, OIL

#### **Random variables**

R.v. as a numerical function on outcomes  
Numerical functions of r.v. are again r.v.

#### **Expectation, variance and sd of r.v.**

$E X = \text{sum of value times probability } \sum_x x p(x)$

$E f(X) = \sum_x f p(f) = \sum_x f(x) p(x)$

$E(aX+bY+c) = aEX + bEY + c$  (always)

$\text{Var}(aX+b) = a^2 \text{Var } X$

$\sigma_{aX+b} = |a| \sigma_X$

$\text{Var}(a X + bY + c) = a^2 \text{Var } X + b^2 \text{Var } Y$  (indep r.v.)

$\sigma_{aX+bY+c} = \sqrt{a^2 \text{Var } X + b^2 \text{Var } Y} = \sqrt{a^2 \sigma_X^2 + b^2 \sigma_Y^2}$  (indep)

$\sigma_{\bar{X}} = \frac{\sigma_{\text{population}}}{\sqrt{n}}$  for independent (e.g. with-repl) samples

Calculations of E (net return) in OIL type problems

Concept of standard score =  $\frac{\text{score} - \text{mean}}{\text{sd}}$

#### **Regression and correlation.**

Scatterplot of (x, y) points in relation to the means of x, y.

Points "upper right" or "lower left" of  $(\bar{X}, \bar{Y})$  give + assoc.

Other points give negative association.

$$\text{Correlation } R = \frac{\overline{xy} - \bar{x}\bar{y}}{\sqrt{\overline{x^2} - \bar{x}^2} \sqrt{\overline{y^2} - \bar{y}^2}}$$

$R(ax+b, cy+d) = R(x, y)$  (if a, c have the same sign)

$R^2$  is the fraction of variance y explained by regression on x \*

\*actually, this is if we use n, not n-1, in calculating variance x

Plot of the regression line thru  $(\bar{X}, \bar{Y})$  with slope  $R \frac{s_y}{s_x}$

Elliptical plot = joint normal

Predicting y from x using regression line\*

\*insert x, read off predicted y from line

### Central Limit (normal) Approximations

Binomial (case when np and nq are large)

n indep trials,

probability p of "success" each trial,

X = number of successes in n trials

$X \sim \text{normal w/ mean } np \text{ and sd } \sqrt{npq}, q = 1 - p$

### Poisson (when it applies)

expect  $\mu$  rare events to occur (i.e. on avg)

sd  $X = \sqrt{\mu}$  (if Poisson applies)

$X \sim \text{normal w/ mean } \mu \text{ and sd } \sqrt{\mu} \text{ if } \mu > 3$

### Sample mean of independent samples from a population

population mean  $\mu$  and sd  $\sigma$ , sample of n

$\bar{X} \sim \text{normal with mean } \mu \text{ and sd } \frac{\sigma}{\sqrt{n}}$

### Difference of independent sample means (large n's)

$\bar{x} - \bar{y} \sim \text{normal}$

mean  $\mu_x - \mu_y$

$$\text{sd } \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$$

### Data calculations

sample mean (balance point of distribution)

sample sd

quartiles

inter-quartile range

box plot

$$a\bar{x} + b\bar{y} + c = a\bar{X} + b\bar{Y} + c$$

s from a list x

$$s_{ax+b} = |a| s_x$$

#### Topics emphasized on Exam 4.

t-based and z-based CI and test for a population mean

t-plot (**requires normal population dist**); getting t from confidence C

z-plot is t-plot for df = infinity\*

\*often used for large df when population dist may not be **normal**

$$\bar{x} \pm t \frac{s}{\sqrt{n}}; \text{ claim made for CI; test statistic } t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}; \text{ df} = n-1$$

H0 and H1; types I and II errors

$$\text{P-value} = 1-C \text{ (2-sided) or } \frac{1-C}{2} \text{ (1-sided)}$$

Reject H0 if P-value <  $\alpha$  = desired type I error rate

z-based CI and test for population proportion p (p is NOT P-value)

$$\hat{p} \pm z \frac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}}; \text{ test statistic } z = \frac{\hat{p} - p_0}{\frac{\sqrt{p_0 q_0}}{\sqrt{n}}} \text{ (notice that } p_0 \text{ is used)}$$

z-based CI and test for  $\mu_x - \mu_y$  based on large indept samples of  $n_x, n_y$

$$\bar{x} - \bar{y} \pm z \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$$

No companion z - test on the exam except questions of interpretation

z-based CI and test for  $\hat{p}_x - \hat{p}_y$  based on large indep samples of  $n_x, n_y$

$$\hat{p}_x - \hat{p}_y \pm z \sqrt{\frac{\hat{p}_x \hat{q}_x}{n_x} + \frac{\hat{p}_y \hat{q}_y}{n_y}}$$

No companion z-test on exam except questions of interpretation.

Margins of error; effect of increasing  $n$

P-value, methods of calculation, use to reach decision on  $H_0$ \*

\*see the prep exercises of Exam 4 for more details

Some prep exercises for the Final Exam will be posted Monday, April 21.