Particle-Filter-Based Multisensor Fusion for Solving Low-Frequency Electromagnetic NDE Inverse Problems

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***Abstract*—Flaw profile characterization from nondestructive evaluation (NDE) measurements is a typical inverse problem. A novel transformation of this inverse problem into a tracking problem and subsequent application of a sequential Monte Carlo method called particle filtering has been proposed by the authors in an earlier publication. In this paper, the problem of flaw characterization from multisensor data is considered. The NDE inverse problem is posed as a statistical inverse problem, and particle filtering is modified to handle data from multiple measurement modes. The measurement modes are assumed to be independent of each other with principal component analysis used to legitimize the assumption of independence. The proposed particle-filter-based data fusion algorithm is applied to experimental low-frequency NDE data to investigate its feasibility.**

***Index Terms*—Data fusion, inverse problems, nondestructive evaluation (NDE), particle filters.**

I. Introduction

THE ESTIMATION of flaw depth profiles (i.e., the sequence of flaw depths as a function of spatial position) from nondestructive evaluation (NDE) measurements is a typical inverse problem. This inverse problem is ill-posed due to nonuniqueness of solutions, particularly in the presence of measurement noise. Various techniques have been proposed in the literature to address ill-posedness [2]. Direct approaches for solving NDE inverse problems typically rely on the use of signal processing techniques to establish a relationship between specific characteristics of a signal and the geometry of a defect, ignoring the underlying physical process. These methods typically pose the inverse problem as determining a mapping from a measurement space to a material property space [3], [4], where the set of unknown parameters that define the mapping are determined from measurements. Direct approaches ranging from calibration methods to more recent procedures based on neural networks [5], [6] have been proposed. The advantages of these approaches are their simplicity and speed; however, the approaches are very sensitive to analytical models that are used as well as noise in measurements. Iterative methods, on the other hand, usually rely on a physical model to accurately simulate the underlying physical phenomenon and predict a probe response [7]. The model is used to estimate measurements given a flaw profile, which is iteratively derived by minimizing the difference between estimated and actual measurements. Minimization may be through conventional techniques such as conjugate gradients, although other techniques such as simulated annealing [8] or genetic algorithms [9] have been proposed. Numerical models such as a finite-element model or integral equation models [3], [6], [9]–[21] have been proposed for electromagnetic NDE signal inversion, and although accurate, they tend to be computationally expensive since the models must be solved iteratively. A Bayesian technique was also proposed for defect signal analysis in NDE images [22], [23], where a hierarchical Bayesian framework was designed for detecting and estimating NDE defect signals from noisy measurements.

The use of multiple inspection modes is becoming common in various NDE applications. Availability of information from multiple measurement modes has the potential for improving accuracy and reliability in flaw profiling due to complementary information contained in multiple sensors. However, this requires development of computationally efficient data fusion techniques for solving inverse problems when multiple measurements are available. Many researchers have employed data fusion techniques to solve the NDE inverse problem [24], [25]. Commonly proposed solutions include neural networks [26]–[29], Bayesian analysis based on the Dempster–Shafer evidence theory [30]–[32], wavelet and other multiresolution algorithms [29], and image fusion [31], [34], [35] in the time-and-frequency domain. These methods have been applied to fuse NDE data from a range of sources, including multifrequency eddy current testing (ECT) [25], [26], [28]; ECT data and ultrasound measurements [26]–[28], [34], [35]; ultrasound, X-ray, and acoustic emission measurements [32]; and other techniques (such as pulsed eddy current measurements) [36]. In addition to these conventional fusion techniques, other methods such as a Q-transform-based technique [37] have also been recently investigated.

Available fusion algorithms are generally based on processing signals or images without regard to the physics of a measurement process. Furthermore, most of the proposed techniques have drawbacks in terms of lower accuracy of

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TABLE I
LIST OF NOTATIONS

Notation	Description
k	Discretized location
x_k	Flaw depth at location k (State)
z_k	NDE measurement at location k
K	Total number of discretized locations
q	Measurement mode
Q	Total number of measurement modes
$\bar{z}_k$	Set of measurements at location k due to all measurement modes
$\vec{Z}$	Set of measurements corresponding to all measurement modes Q at all locations K
$\vec{X}$	Set of states (flaw depths)
N_k	Elements in the neighborhood of location k
L	Parameter controlling size of neighborhood
x_k^i	i^{th} Sample (particle) at location k
$w_k^{i,q}$	Weight assigned to x_k^i by qth measurement mode
N_s	Total number of samples (particles)
I	No. of iteration
τ	Preset convergence threshold
c	Measurement model r-th coefficient
R	Order of polynomial measurement model
$x_{k\|k}^{MMSE}$	Posterior mean estimate
$x_{k\|k}^{MAP}$	Maximum a posteriori estimate
MSE	Mean square error
RMSE	Root Mean square error

inversion and high computational cost. In order to address these drawbacks, the authors have proposed a sequential Monte-Carlo-based method for the solution of low-frequency NDE inverse problems [1]. In this paper, the sequential Monte Carlo technique is extended to fusing NDE data from multiple NDE measurement modes.

The rest of this paper is organized as follows. The next section describes the problem formulation for an NDE inverse problem in terms of a recursive framework in the presence of measurement data from multiple measurement modes. A particle filtering technique followed by its application for solving the NDE inverse problem in the presence of multiple measurement modes is discussed. The particle-filter-based data fusion technique is developed assuming that NDE measurement modes are uncorrelated to each other. The use of principal component analysis (PCA) to legitimize the assumption of independence of measurement modes is then discussed. Results of flaw profiling from multimodal NDE measurements are presented to validate the proposed techniques. A comparative study of flaw profiling results when using a single-measurement mode and data fusion with/without PCA is also reported in this paper. Finally, conclusions and future work are presented. The notations used in this paper are tabulated in Table I for clarity.

II. NDE INVERSE PROBLEM FORMULATION

A. Problem Formulation

The problem formulation described here is applicable to flaw profile reconstruction in both 2-D and 3-D. The 2-D problem

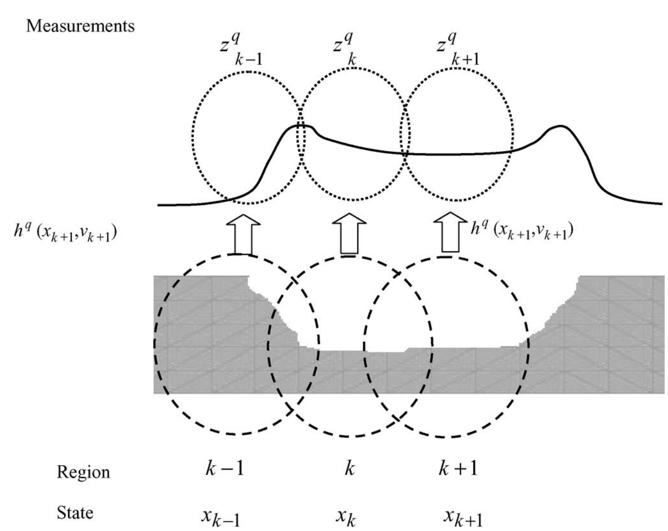

Fig. 1. NDE inverse problem for 1-D flaw profile reconstruction (assuming $L = 0$).

is equivalent to estimating flaw depths along the length of a specimen (where the depth along the width dimension is assumed to be invariant). In this case, the length of the specimen is divided into K locations. The 3-D problem is equivalent to estimating flaw depths at each location on a specimen surface; therefore, the surface of the specimen is divided into K discrete locations. In each case, flaw depth is unknown at each discretized location. Examples of the formulation for 2-D (cracklike) and 3-D (volumetric flaws such as corrosion, wear, or wall thickness loss) profiling are shown in Figs. 1 and 2,

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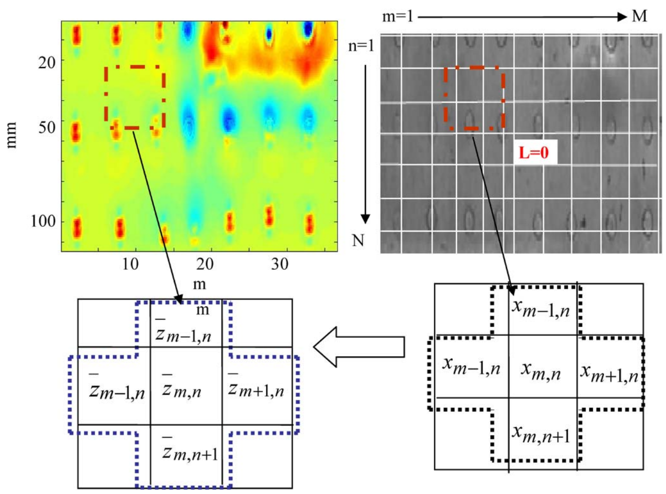

Fig. 2. NDE inverse problem for 2-D flaw profile reconstruction (assuming $L = 0$).

respectively. The depth profile of flaws in a specimen (2-D or 3-D) is expressed in terms of set $\vec{X} = \{x_1, x_2, \ldots, x_{k-1}, x_k, x_{k+1}, \ldots, x_{K-1}, x_K\}$, where each element x_k of the set is stated (flaw depth) at the discrete location k. Assume that measurements $\bar{z}_k = \{z_k^1, z_k^2, \ldots, z_k^q, \ldots, z_k^Q\} = \{z_k^q | q = 1 : Q\}$ from Q measurement modes are available at each position index k ($1 \le k \le K$). The evaluation of the sequence of states given the set of measurements is the inverse problem. This inverse problem can be formulated in terms of a statistical estimation problem. In this case, the unknown parameter is the set of states $\vec{X}$, whereas the corresponding observation is the set of measurements $\vec{Z} = \{z_1^1, z_1^2, \ldots z_1^Q, z_2^1, \ldots z_k^q, \ldots z_K^Q\}$. The posterior probability density function (pdf) $p\vec{X}|\vec{Z}$ of states $\vec{X}$ for each measurement mode q can be computed in the framework of statistical inverse problems [38]. States $\vec{X}$ are estimated from the posterior pdf as follows:

$$p(\vec{X}|\vec{Z}) \propto p(\vec{Z}|\vec{X})p(\vec{X}). \tag{1}$$

Here, $p(\vec{Z}|\vec{X})$ is the likelihood function, and $p(\vec{X})$ captures available prior information about the set of states (set of flaw depths). The estimation of $p(\vec{Z}|\vec{X})$ is generally a computationally expensive process due to the high dimensionality of the state vector $\vec{X}$. Evaluating the posterior pdf sequentially (i.e., at each position by stepping through the position index k) can potentially keep the computational cost and the complexity of the inverse problem low. Three conditions, which are fundamental to the proposed problem formulation, are assumed to enable a sequential solution to the inverse problem [38]. First, a locally dependent Markov field is adopted to solve the problem. This Markovian assumption has been shown to be valid for low-frequency electromagnetic NDE (e.g., magnetostatic and eddy currents) [39]. Therefore, the relationship offered by (1) holds locally, and the equation can also be written for neighborhood N_k around the position index k, where $1 \le k \le K$, as follows:

$$p\left(x_k|\{\bar{z}_j|j \in N_k\}\right) \propto p\left(\{\bar{z}_j|j \in N_k\}|x_k\right) p(x_k). \tag{2}$$

The neighborhood elements N_k of each discretized location for the 2-D problem are given by the 1-D window $x_{k-(2L+1):k-1,\,k+1:k+(2L+1)}$, as shown in Fig. 1 L is a scalar parameter, which controls the size of the neighborhood. The neighborhood elements N_k for each location for a 3-D problem are defined by a 2-D window on the surface of specimen $x_{p,\,q}\big|_{\substack{p=m-(2L+1):m-1,\,m+1:m+(2L+1),\\ q=n-(2L+1):n-1,\,n+1:n+(2L+1)}}$, as shown in Fig. 2. The neighborhood, as defined here, is similar to the lexicographic arrangement of pixels commonly used in the image processing literature [40].

The second assumption is that the observations (or measurements) are mutually independent within the neighborhood of the state, given the true values of the unknown states as

$$p\left(\{\bar{z}_j|j \in N_k\}|x_k\right) = \prod_{j \in N_k} p(\bar{z}_j|x_j). \tag{3}$$

Finally, the prior density of the unknown state is assumed to be a product of exponential densities centered on the value of the states in the neighborhood as follows:

$$p\left(x_k|x_j|_{\substack{j\in N_k\\ j\neq k}}\right) = e^{-\sum\limits_{j\in N_k} \frac{\|x_j - x_k\|^p}{p}} \tag{4}$$

where p is a scalar, with a value selected to be around 1 to ensure a low amount of variability in the state value within the neighborhood, and $x_j|_{\substack{j\in N_k\\ j\neq k}}$ are the states in neighborhood N_k of location k. The assumptions specified by (3) and (4) simplify the computation for local Markov fields [38].

B. *Inverse Problem as a Tracking Problem*

As discussed earlier, the inverse problem in NDE is to determine the best flaw characteristics that match the measurements. The inverse problem may be modeled using two sets of equations [41]: A state transition equation that is used to model the evolution of the states with respect to spatial position, given as

$$x_k = f_k\left(x_j|_{\substack{j\in N_k\\ j\neq k}},\, \nu_k\right) \Leftrightarrow p\left(x_k|x_j|_{\substack{j\in N_k\\ j\neq k}}\right) \tag{5}$$

and measurement models that relate the measurements from multiple sensors to the states at a given position, given as

$$z_k = h_k^q\left(x_k,\, \mu_k^q\right) \Leftrightarrow p(z_k|x_k) \tag{6}$$

where f_k and h_k^q are functions that model the state transition and measurement processes, respectively, and ν_k and μ_k^q represent the process noise and the measurement noise, respectively. Note that the state transition and measurement equations are applied to the local neighborhood of state x_k. The problem described by (5) and (6) is often referred to as a tracking problem and is used frequently in target tracking applications [41], [42]. In these applications, the state transition function models the motion of a target from the known position x_j, whereas the measurement function describes some function of the target position. Process and measurement noise densities represent the uncertainty in the state and measurement models. The tracking problem, as defined in (5) and (6), is a dynamic state estimation problem [41]. A Bayesian approach to this

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problem is to construct a posterior pdf of the state, based on the sequence of measurements.

The problem described by (5) and (6) can be shown to be equivalent to the statistical inverse problem defined by (1) using the following arguments. The state transition function (when taken with the associated process noise distribution) is equivalent to the local prior pdf used in the statistical inverse problem. Similarly, the measurement function is equivalent to the local likelihood pdf in the statistical inverse problem when a locally independent Markov field is assumed (see Section II-A). The equivalence of the tracking problem to statistical probabilities is also shown in (5) and (6). The advantage of this equivalence is the potential applicability of solution techniques for tracking problems to solve the NDE inverse problem. Note also that the formulation of the inverse problem as the statistical inverse problem (1) or, equivalently, a tracking problem, as in (5) and (6), implicitly assumes that the NDE measurement process is a localized process; i.e., the measurement at the particular location k is only affected by the state of the sample in a neighborhood of k. As aforementioned, the Markovian assumption is valid for low-frequency electromagnetic NDE (magnetostatic and eddy current NDE) [40], provided that the neighborhood is selected appropriately.

III. Application of Particle Filters to Solve NDE Inverse Problems Using Multiple-Mode Measurement Data

A. Solutions to the Tracking Problem

Kalman filtering [41] provides an optimal solution to the tracking problem if the following two conditions are met.

1) The function that relates the states in a neighborhood (i.e., the state transition function) and the function that relates the state to measurement (i.e., the measurement function) are linear.
2) The likelihood and prior pdfs are Gaussian.

In general, for the NDE inverse problem, these functions can be nonlinear, and the pdfs can be non-Gaussian (multimodal). Therefore, the Kalman filter cannot provide an optimal solution, and suboptimal algorithms may be necessary to evaluate flaw depth. Therefore, a more generalized filtering technique is required to solve this inverse problem. Particle filters offer such a generalized filtering technique.

B. Theory of Particle Filters

Particle filters are sequential Monte Carlo methods based on point-mass (or "particle") representations of probability densities that can be applied to any state-space model and that generalize traditional Kalman filtering methods [41], [42]. In this approach to a dynamic state estimation, the posterior pdf of the state is constructed based on all available information, including the set of received measurements. A brief description of the particle filter algorithm is provided next. To simplify the discussion, we assume that measurements $z_k = \{z_k^q | q = 1\} \equiv z_k$ from a single-measurement mode (i.e., $Q = 1$) are available. This assumption will be relaxed in the next section, where we extend the particle filtering framework to incorporate multimodal measurements.

With the assumption of a single-measurement mode (and temporarily simplifying the notation by dropping superscript q), the pdf of state x_k conditioned on all measurements up to (and including z_k) $p(x_k,\, z_{1:k})$ may be obtained recursively in two stages: prediction and update stages. The prediction stage uses the system model to predict the pdf forward from one measurement location to the next. Suppose that the required pdf $p(x_k|z_{1:k-1})$ at location $k-1$ is available. The prediction stage involves using the system model to obtain the prediction density of the state at k via the Chapman–Kolmogorov equation [41] as follows:

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1}) p(x_{k-1}|z_{1:k-1}) dx_{k-1}. \tag{7}$$

If we consider a Markov process of order 1, then $p(x_k|x_{k-1},\, z_{1:k-1}) = p(x_k|x_{k-1})$. Since the state is usually subject to unknown disturbances (modeled as random noise), the prediction step generally translates, deforms, and otherwise distorts the pdf. The update operation uses the latest measurement to modify the prediction pdf. This is achieved using Bayes' theorem as follows:

$$\begin{aligned} p(x_k|z_k) &= p(x_k|z_k, z_{1:k-1}) \\ &= \frac{p(z_k|x_k,\, z_{1:k-1}) p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} \\ &= \frac{p(z_k|x_k) p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} \end{aligned} \tag{8}$$

where the normalizing constant is given by

$$p(z_k|z_{1:k-1}) = \int p(z_k|\vec{X}) p(\vec{X}|z_{1:k-1}) dx_k. \tag{9}$$

In order to apply particle filtering, the desired posterior pdf is represented in terms of samples and associated weights at each location. In order to develop the details of the algorithm, let $\{x_k^i,\, w_k^i\}|_{1:N_s}$ denote a random measure that characterizes the posterior pdf at location k. x_k^i is the set of support points with associated weights w_k^i, and $i = 1 : N_s$ is the total number of samples used. The weights are normalized such that $\sum_{i=1}^{N_s} w_k^i$. Then, posterior density at k can be approximated as [41]

$$p(x_k|z_k) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_k - x_k^i\right). \tag{10}$$

Normalized weights are chosen using the principle of importance sampling [41]. According to this principle, suppose that $p(x) \propto \pi(x)$ is a probability density from which it is difficult to draw samples but for which $\pi(x)$ can be evaluated, and samples can be drawn from $\pi(x)$. In addition, let x^i be the samples that are easily generated from proposal $q(.)$ called importance density. Then, a weighted approximation to density $p(x)$ is given by

$$p(x) \approx \sum_{i=1}^{N_s} w^i \delta(x - x^i) \tag{11}$$

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where $w^i \propto (\pi(x^i)/q(x^i))$ is the normalized weight of the ith particle. Therefore, if samples x_k^i were drawn from the importance density $q(x_{1:k}|z_{1:k})$, the weights are given by [41]

$$w_k^i \propto \frac{p\left(x_{1:k}^i|z_{1:k}\right)}{q\left(x_{1:k}^i|z_{1:k}\right)}. \tag{12}$$

With the reception of measurement z_k at position k, we wish to approximate $p(x_{1:k}|z_{1:k})$ with a new set of samples. Given the set of weights w_{k-1} at position $k-1$, the weights at position k may be computed recursively using the weight update equation derived from the principle of importance sampling as

$$w_k^i \propto w_{k-1}^i \frac{p\left(z_k|x_k^i\right) p\left(x_k^i|x_{k-1}^i\right)}{q\left(x_k^i|x_{k-1}^i, z_k\right)}. \tag{13}$$

The most commonly used variant of a particle filter, the sampling importance resampling (SIR) algorithm [42], is used in this paper. The importance density in the SIR algorithm is chosen as the transition prior to

$$q\left(x_k^i|x_{k-1}^i, z_k\right) = p\left(x_k^i|x_{k-1}^i\right). \tag{14}$$

Therefore, from (12) and (13), the following is derived:

$$w_k^i \propto w_{k-1}^i p\left(z_k|x_k^i\right). \tag{15}$$

We can also write (15) as

$$w_k^i \propto p\left(z_k|x_k^i\right). \tag{16}$$

C. Particle Filtering for Multisensor Data Fusion

When multiple measurement modes are available, likelihood pdfs corresponding to each measurement mode need to be considered in weight assignment to the sample. Assume that $w_k^{i,q}$ is the weight of the sample i at the position index k assigned by the individual measurement mode q. For every sample at each position index, Q weights are therefore computed using the respective likelihood pdfs. The likelihood function corresponding to the qth measurement mode is given by (16) and the following:

$$w_k^{i,q} \propto p\left(z_k^q|x_k^i\right). \tag{17}$$

If the measurement processes are assumed to be independent, then the joint likelihood due to measurement modes $q = 1, 2, \ldots, Q$ is the product of likelihood for the individual measurement mode, given by

$$p\left(z_k|x_k^i\right) = p\left(z_k^1|x_k^i\right), p\left(z_k^2|x_k^i\right), \ldots, p\left(z_k^Q|x_k^i\right). \tag{18}$$

Therefore, from (17) and (18), we get the following:

$$w_k^i \propto p\left(z_k^1|x_k^i\right).p\left(z_k^2|x_k^i\right)\ldots p\left(z_k^Q|x_k^i\right). \tag{19}$$

Using (16) and (19), the final weight assigned to sample i at position index k is as follows:

$$w_k^i \propto w_k^{i,1}, w_k^{i,2}, \ldots, w_k^{i,Q}. \tag{20}$$

As aforementioned, it is assumed that the measurement processes are independent. However, the measurement processes may be correlated, and the assumption of independence is not valid in that case. In order to make the assumption of independence more legitimate, the PCA technique is applied to data from different measurement modes. PCA [43] is mathematically defined as an orthogonal linear transformation that transforms the data to a new coordinate system. PCA can also be applied to data from multiple measurement modes. At each position index k, measurements $z_j^{1:Q}|_{j\in N_k}$ within its neighborhood from all measurement modes $(1:Q)$ are considered. These multidimensional data are now the input of the PCA technique. The following steps are carried out to evaluate the principal components.

Step 1: The multisensor measurements $z_j^{1:Q}|_{j\in N_k}$ are stored as vector $\varsigma^q|_{q=1:Q}$, where each measurement mode is assumed to be one component of the vector.

Step 2: The data vector is adjusted by subtracting out its mean given by:

$$\varsigma^q = \varsigma^q - \mathrm{mean}(\varsigma^q). \tag{21}$$

Step 3: The adjusted data vectors are arranged as rows of a matrix. This newly formed matrix will be called the "adjusted data matrix," given as

$$\varsigma = [\varsigma^1, \varsigma^2, \ldots, \varsigma^Q]. \tag{22}$$

Step 4: The covariance matrix of the "adjusted data matrix" is computed as follows:

$$\sigma^q = \mathrm{cov}(\varsigma^q). \tag{23}$$

Step 5: Eigenvectors λ^q of the covariance matrix are then evaluated as follows:

$$\lambda^q = \mathrm{eig}(\sigma^q). \tag{24}$$

Step 6: The computed eigenvectors are arranged as rows of a new matrix. This newly formed matrix will be referred to as the "feature matrix" as follows:

$$\lambda = [\lambda^1, \lambda^2, \ldots, \lambda^Q]. \tag{25}$$

Step 7: Finally, the "feature matrix" is multiplied by the "adjusted data matrix"

$$\psi = \lambda\varsigma. \tag{26}$$

The rows of the resultant matrix ψ are the principal (uncorrelated) components in the data as follows:

$$\psi = [\psi^1, \psi^2, \ldots, \psi^Q]. \tag{27}$$

The resultant components ψ^q are uncorrelated to each other. Therefore, the output of the PCA technique is a set of independent (uncorrelated) data of Q dimensions. These independent components are then treated as the data from separate measurement modes.

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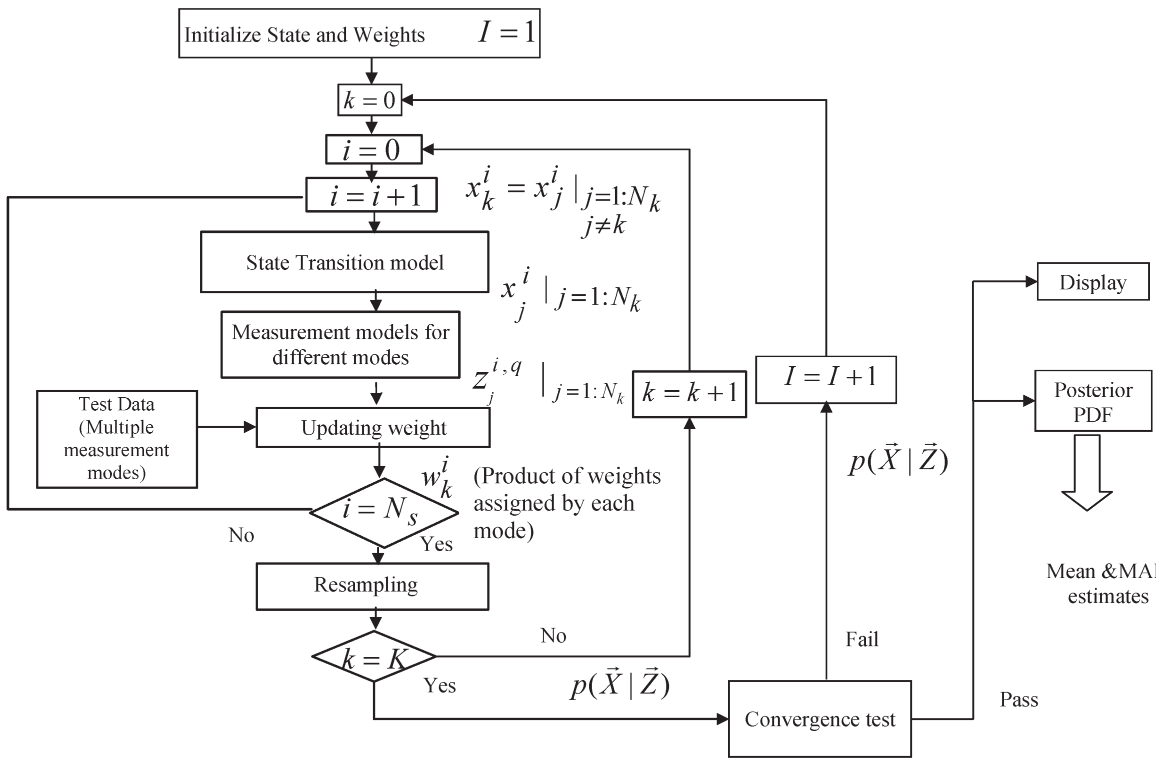

Fig. 3. Particle filter implementation for flaw profiling.

D. Implementation

The sequence of steps for evaluation of the posterior pdf of the state (flaw depth) is shown in Fig. 3 and is summarized below.

Step 1: In the initialization, N_s samples are sampled at each position index from the prior pdf, as given in (4).

Step 2: In weight assignment, weights are assigned using the likelihood pdf, as indicated in (15). The likelihood pdf is represented by the error between the computed measurement using the measurement model and the actual measurement. As shown in Fig. 3, actual measurements (test data) and the computed measurement (using a measurement model) are inputs to the weight assignment block. If the difference between computed and actual measurements for a sample (particle) is small, the likelihood or weight of a sample is high and vice versa. The measurement model typically relates the state to measurements, and the results of inversion may be expected to depend on the choice of the measurement model. The measurement model is derived from a training database of known states and corresponding NDE measurements. A comparative study using different measurement models to establish a relationship between the state and low-frequency electromagnetic NDE measurements, in terms of accuracy of inversion and computational load, was carried out [44]. Based on the results reported in [44], the measurements (response) are approximated from the state through an Rth-order polynomial. The model is expressed as follows:

$$z_k = \sum_{r=0}^{R} c_r x_k^r. \tag{28}$$

The coefficients of polynomial c are determined from a training database of known states and corresponding measurements.

Step 3: In resampling, a common problem with particle filters is degeneracy [42], where after a few iterations, all but one particle will have negligible weight. The basic idea of resampling is to eliminate particles that have small weights and concentrate on particles with large weights. The posterior pdfs (samples and associated weights) are estimated at all locations.

Step 4: In convergence checking, single-point estimates of the unknown state are computed from the evaluated posterior pdf, as described in Section IV-A. The estimated posterior pdfs at all locations $(1 : K)$ are then used as initial values and are updated iteratively (see Fig. 3). In subsequent iterations, the states (flaw depths) in the neighborhood of each location are the single-point estimates from the pdfs evaluated at preceding iterations. The algorithm runs iteratively until the error between the single-point flaw profile estimates in two consecutive iterations is less than some preset convergence criterion. If each iteration is denoted by I and the estimated flaw depths throughout the specimen (K discretized locations) evaluated at I is $x_{1:K}(I)$, then the convergence criterion is given by

$$\frac{\sum_{k=1}^{K} (x_k(I+1) - x_k(I))^2}{K} \leq \tau. \tag{29}$$

Here, τ is the preset convergence threshold. At convergence, $x_{1:K}(I+1)$ is assumed to be the predicted flaw profile.

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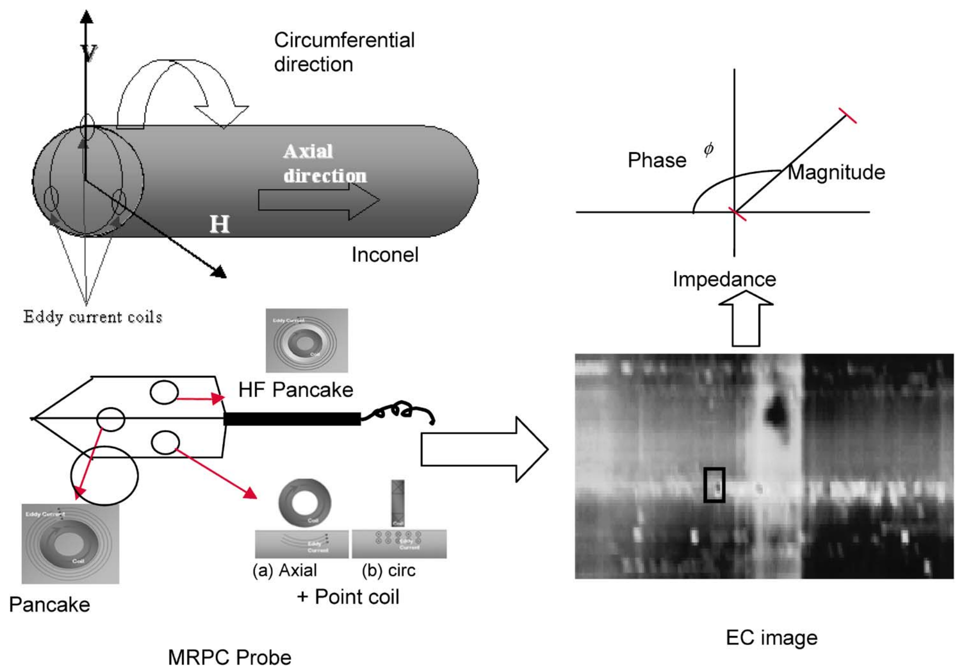

Fig. 4. ECT of steam generator tubing.

The assumption of independence of measurement modes offers the foundation to the proposed particle-filter-based data fusion technique. As the output of the PCA technique is a set of uncorrelated components, this transformation gives legitimacy to the assumption that measurement data is independent.

IV. Results

The proposed particle-filter-based technique was applied to steam-generator (SG)-tubing NDE data for flaw profiling as well as for corrosion quantification in aircraft lap joints. Evaluation measurements are used to evaluate the performance of the proposed inversion algorithm.

A. Evaluation Measurements

As discussed in the previous sections, the posterior pdf of the unknown quantity is evaluated at discrete positions using the proposed technique. Posterior mean estimates (PME) and maximum *a posteriori* (MAP) estimates of the states at each location are subsequently computed from the posterior pdfs. These computed estimates are assumed to be the predicted flaw profiles in this paper. The posterior mean estimate is given by

$$\hat{x}_{k|k}^{\mathrm{MMSE}} = E\{x_k|z_k\} = \int x_k p(x_k|z_k) dx_k \tag{30}$$

whereas the MAP estimate is given as

$$\hat{x}_{k|k}^{\mathrm{MAP}} = \max_{x_k} p(x_k|z_k). \tag{31}$$

The resulting predicted profiles using the PME and MAP estimates were compared with the true profiles using a mean-square-error (MSE) measurement [1] as follows:

$$\mathrm{MSE} = \frac{\sum_{k=1}^{K} \left(x_{k(\mathrm{predicted})} - x_{k(\mathrm{actual})}\right)^2}{K}. \tag{32}$$

B. SG Tubing NDE

The proposed technique is applied to estimate flaw profiles from a multifrequency eddy current inspection [45] of SG tubing in nuclear power plants.

i) Data Description: The experimental setup for ECT is shown in Fig. 4 [45]. The setup includes an SG tube along with a motorized rotating probe coil (MRPC). The MRPC probe contains three coils: a plus (“+”) point coil, a pancake coil, and a high-frequency pancake coil. Measurements were acquired on tubes with laboratory-induced cracking as well as tubes with cracking pulled from operational power plants. After the NDE measurements were acquired, the tubes were destructively analyzed, and the resulting metallographic results documenting the flaw depth profiles were used as “ground truth” in this paper. Selected flaws from this database were used to formulate the training database from which the measurement model parameters were estimated. For evaluating the proposed algorithm performance, the magnitude and the phase of the complex eddy current data from the MRPC plus (“+”) point probe, with excitation frequencies of 100, 200, and 300 kHz, were used as the measurements.

ii) Results: Twelve different defect profiles were selected from an industry test database for investigating the efficacy of the proposed inversion technique. The flaw profiles are tabulated in Table II. Only the width and maximum depth are tabulated in the table. The number of samples used in the particle filter algorithm, at each position index, was 2000, whereas

TABLE II
EXPERIMENTAL FLAW PROFILES

Flaw ID	1	2	3	4	5	6	7	8	9	10	11	12
Width (mm)	2.75	7.00	5.50	3.00	2.75	2.25	5.25	11.00	2.50	3.00	3.25	3.50
% Max Depth	20	100	100	85	90	70	78	60	58	71	39	76

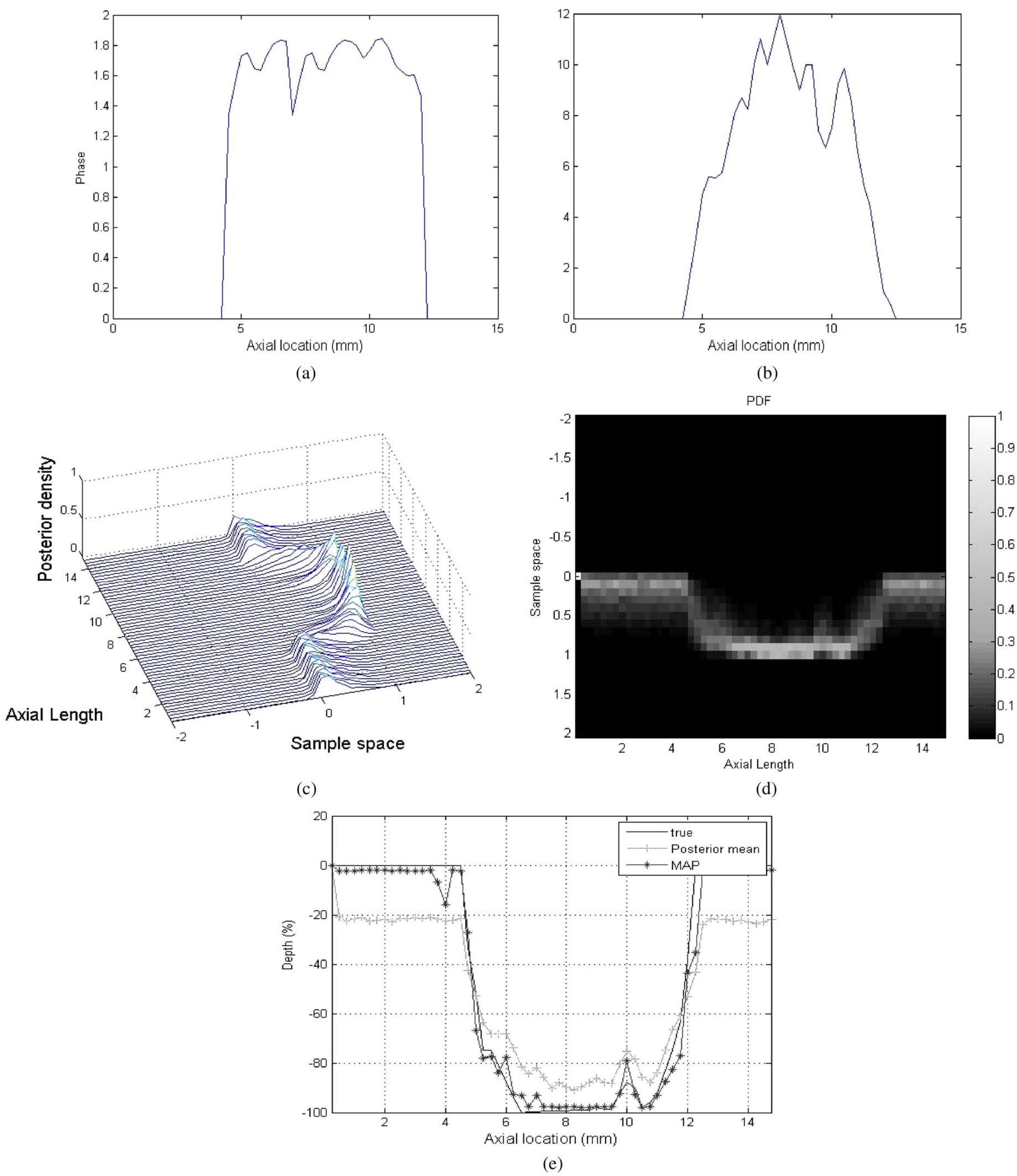

Fig. 5. Proposed inversion technique. (a) Phase measurement. (b) Magnitude measurement. (c)Three-dimensional and (d) top views of the posterior pdf. (e) Computed estimates.

a third-order polynomial measurement model was used. Three different values of parameter L (0, 1, and 2) were used. The convergence threshold used in this paper was $\tau = 10^{-3}$. Fig. 5 shows the inversion results using data fusion of magnitude and phase measurement at 300 kHz for a flaw with a width of 7 mm and a maximum depth of 100% of tube wall thickness. Fig. 5(a)

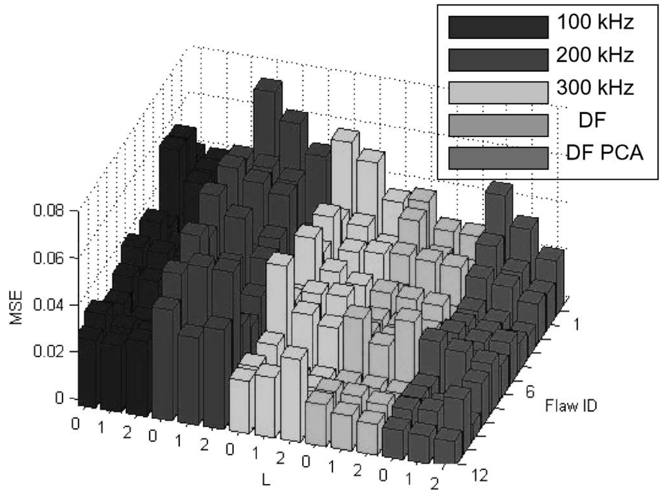

Fig. 6. MSE between the PME of the posterior pdf and the true profiles versus the state vector length parameter L at different measurement modes for experimental data.

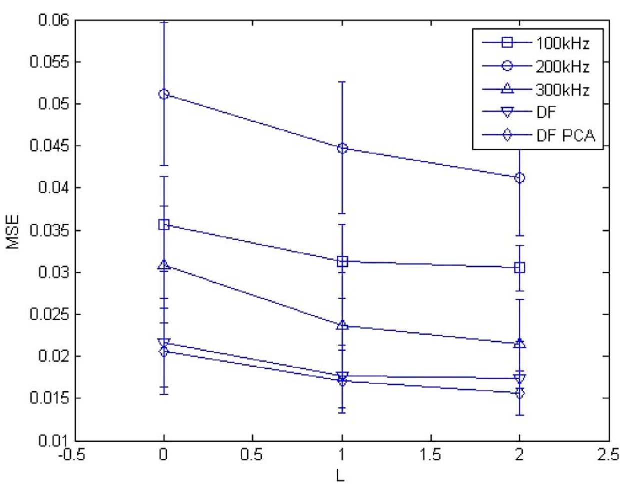

Fig. 7. Mean and standard deviations of the MSE (between the PME and the true profile) versus L at different measurement modes.

and (b) shows phase and amplitude measurements, respectively. Fig. 5(c) and (d) shows the top and 3-D views of the computed posterior pdfs, respectively. Fig. 5(e) shows the computed estimates.

Fig. 6 presents a summary of the MSE between the PME and true flaw profiles for the 12 flaws, using different measurement modes and the state vector length parameter $L = 0, 1, 2$. The MSE between the PME and the true flaw profile using measurement data at three individual frequencies (100, 200, and 300 kHz) are shown. The MSE between the PME and true profiles using data fusion of measurement modes without PCA and with PCA are shown in Fig. 6. The average and standard deviation of the MSE for each measurement mode are shown in Fig. 7.

C. Corrosion Quantification in Aircraft Lap Joints

i) Data Description: Quantification of the loss of plate thickness due to corrosion on an aircraft skin is presented as a test case for 3-D flaw profile construction. The NDE

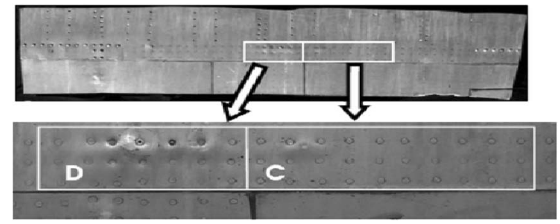

Fig. 8. Aircraft lap joint specimen.

measurement data was acquired from a 30-year-old service-retired Boeing 727 aircraft, as shown in Fig. 8. Two specimens (C and D), each consisting of a two-layer aluminum 2024-T3 lap joint cut out from below the cargo floor, were inspected. The thickness of each layer of the lap joint is 0.045 in (1.143 mm). The specimens were disassembled and cleaned of all corrosion products after the inspection. X-ray mappings were then acquired to assess the true plate thickness. The inversion results were compared with the true thickness to determine the efficacy of the technique. Measurements from multiple-frequency eddy current modes (5.5, 17, and 30 kHz) were used in the inversion. The number of samples N_s in the particle filter was 2000. The resulting predicted profiles using PMEs and the MAP estimates are compared with the true profiles using a root MSE measurement [46].

ii) Results: The inversion result evaluated using a single-measurement mode and fusing multiple measurement modes for specimen C is shown in Fig. 9. Results for specimens C and D are tabulated in Table III. Results indicate improvement in inversion accuracy when data fusion is used.

D. Discussion

Flaw depth profiling results from eddy current measurements indicate that the proposed particle filter approach is capable of producing reasonably accurate results even in the presence of noise. In particular, results on experimental data validate the robustness of the proposed approach and indicate the feasibility of the proposed inversion technique. The use of data fusion is also seen to reduce the error in the reconstruction when compared to flaw profiles obtained using a single-measurement mode. The use of PCA further increases the accuracy of inversion. Note that in all cases, the flaw profile reconstruction processes were obtained using the particle-filtering algorithm. Comparison with other contemporary techniques reported in the literature [46] indicates that the inversion results using particle filtering (with or without data fusion) are either comparable to or better than the results obtained using these other techniques. The results also indicate that using a large neighborhood improves the inversion results. Note that the depth variation in real flaws can be abrupt, and flaw depth does not remain constant for long spans. This fact is a challenge for the proposed inversion technique. The use of a larger neighborhood (large value of L) appears to smooth out the variations in the flaw profiles and improves inversion accuracy. However, increasing the size of the neighborhood increases the computational cost. For instance, increasing L from 0 to 2 increases the computational load by a factor of 5 [47].

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

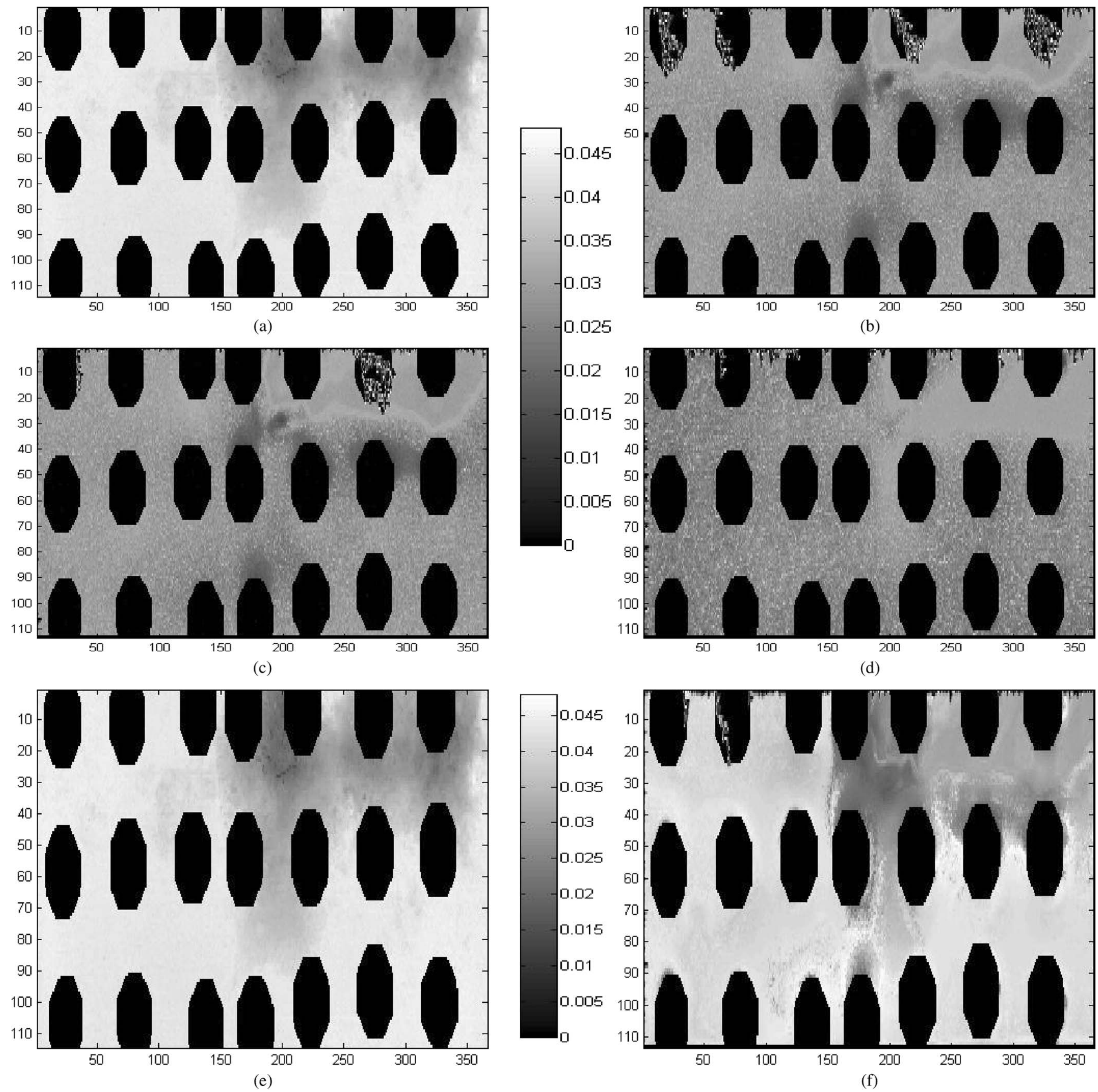

Fig. 9. Inversion results of specimen C (data fusion). (a) and (e) True profile. (b)–(d) and (f) Predicted profiles using ET data at 5.5, 17, 30 kHz and data fusion.

TABLE III
INVERSION RESULTS FOR SECTIONS C AND D

S.No	Measurement mode	RMSE (X10-3)	
		Section C	Section D
1.	17 kHz Eddy Current	3.19	4.31
2.	30 kHz Eddy Current	2.96	3.82
3.	P-ET LOI	3.89	5.13
4.	Data Fusion (17kHzET+30kHzET+P-ET LOI)	1.86	2.21

V. CONCLUSION & FUTURE WORK

A sequential Monte-Carlo-based data fusion technique was proposed for solving inverse problems. Results on multiple databases indicate the efficacy of the proposed inversion technique. The proposed data fusion technique is based on the assumption that measurement processes are statistically independent processes, and the use of PCA was proposed to further legitimize the assumption. The results indicate that the inversion results improve when data from multiple measurement modes are fused. A further improvement is observed in the inversion results when PCA is used prior to data fusion. Future work will focus on investigation of alternative preprocessing approaches when the assumption of independence is not valid. Application of the proposed technique on other types of data sets is another focus area for future work. These data sets include X-ray tomography and ultrasonic data.

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