STT995 Topics on Fractional Brownian Motion
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Prerequisite: STT882 or permission of instructor.

This course is on probabilistic, geometric and statistical properties of *self-similar processes* (i.e., invariant under scaling). Many of the stochastic processes that we will discuss have the properties of *long-range dependence* (also called long memory) and/or *heavy tailedness*. A typical example of such processes is *fractional Brownian motion*. In recent years there have been a vast number of applications of these processes to various areas. In particular, we will discuss their applications in modelling telecommunication network traffic and insurance risk.

The main objective of this course is to equip the students with a large family of probabilistic models (i.e., self-similar processes) and with powerful mathematical tools to analyze them probabilistically and statistically. Such tools include general Gaussian principles, fractal dimensions, wavelet analysis, and so on. As applications of the theory, we will study the overflow probability in telecommunication networks and the ruin probability in insurance risk theory.

This is a vast and developing area with many interesting applications. There are many open questions of either probabilistic or statistical nature. Some of these problems are appropriate for Ph.D. thesis.

The following is a tentative list of the topics that will be covered:

1. Introduction on Self-Similar Processes: General properties and classes of self-similar processes; Lamperti's theorem.

2. Fractional Brownian Motion (fBM) and Its Properties: Stochastic integral representations of fBM; Long range dependence; Weak convergence to fBM: non-central limit theorems and correlated random walks; Distribution of the maximum: large deviations and small ball probabilities; Modulus of continuity and laws of the iterated logarithm; Fractal dimensional properties.

3. Applications of fBM in Network Traffic and Insurance Risk Modelling: The On/Off model and the infinite source Poisson model; Interaction between heavy-tailedness and long range dependence; Approximation of the workload (or risk) process by fBM; Estimation of the overflow (or ruin) probability.

4. Statistical Analysis of Fractional Brownian Motion: Wavelet transforms of fractional Brownian motion; Estimating the Hurst index using wavelet method and/or the fractal dimension.

If time permits, we will discuss several extensions of fBM such as multifractional Brownian motion and some self-similar stable processes.