7.7.1) a) $\mathbb{P}(X \leq u) = 1 - e^{-\frac{u}{\theta}}$

$$\Rightarrow \mathbb{P}\left(\theta \geq -\frac{X}{\ln(1-\alpha)}\right) = \alpha$$

$$\Rightarrow \left(0, -\frac{X}{\ln(1-\alpha)}\right] \text{ is a } 100(1-\alpha)\% \text{ upper CI for } \theta$$

b) $X_i \overset{i.i.d.}{\sim} \mathcal{E}(1/\theta) \Rightarrow \frac{2}{\theta}\sum_{i=1}^{10} X_i \sim \chi^2_{20}$

$$\Rightarrow \mathbb{P}\left(\chi^2_{20,\alpha/2} \leq \frac{2}{\theta}\sum_{i=1}^{10} X_i \leq \chi^2_{20,1-\alpha/2}\right) = 1-\alpha$$

$$\Rightarrow \mathbb{P}\left(\frac{2\sum X_i}{\chi^2_{20,1-\alpha/2}} \leq \theta \leq \frac{2\sum X_i}{\chi^2_{20,\alpha/2}}\right) = 1-\alpha.$$

c) $(6.004, 17.382)$

7.7.2) $f(x;\eta) = e^{-(x-\mu)} \cdot I_{\{x > \eta\}}$

$\Rightarrow X_{(1)} = \min\{X_1, \dots, X_n\}$ is the MLE for η.

$$\Rightarrow \mathbb{P}\left(\eta \geq X_{(1)} + \frac{1}{n}\ln(\alpha)\right) = 1-\alpha$$

$\Rightarrow [12.957, \infty)$ is the 95% lower CI for η.

7.7.4) a) $\mathbb{P}\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = .95$

$$\Rightarrow 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .03$$

$$\xRightarrow{\hat{p}=.2} n_w = 683$$

$$\Longrightarrow n_{w_0} = \frac{n_w}{1 + \frac{n_w}{N}} \simeq 493$$

7.7.4) b) $(\hat{p}-p)/\sigma \sim N(0,1)$ where

$$\sigma = \sqrt{\frac{p(1-p)}{n} \cdot \frac{N-n}{N-1}}$$

$$\Rightarrow \quad \mathbb{P}(|\hat{p}-p| \geq a) = 2(1-\Phi(a/\sigma))$$

c) $\mathbb{P}\left(\frac{\hat{p}-p}{\sigma} > z_\alpha\right) = 1-\alpha$

$$\Rightarrow \mathbb{P}\left(p \leq \hat{p} + z_{1-\alpha} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n} \cdot \frac{N-n}{N-1}}\right) = 1-\alpha$$

for $\alpha = .05$, $\hat{p} = \frac{47}{500} = .094 \Rightarrow (0, 0.116)$ CI 95% for p.

7.7.9) a) $A_i := \{\theta_i \in I_i\}$; $i = 1, 2, \sim, k$

We know: $\mathbb{P}(A_i) = 1-\alpha_i \Rightarrow \mathbb{P}(A_i^c) = \alpha_i$

$$\Rightarrow \mathbb{P}\left(\bigcup_{i=1}^{k} A_i^c\right) \leq \sum_{i=1}^{k} \mathbb{P}(A_i^c) = \sum_{i=1}^{k} \alpha_i$$

$$\Rightarrow \mathbb{P}\left(\bigcap_{i=1}^{k} A_i\right) = 1 - \mathbb{P}\left(\bigcup_{i=1}^{k} A_i^c\right) \geq 1 - \sum_{i=1}^{k} \alpha_i$$

b) $$\mathbb{P}\left(\bigcap_{i=1}^{k} A_i\right) = \prod_{i=1}^{k} \mathbb{P}(A_i) = \prod_{i=1}^{k} (1-\alpha_i)$$

7.7.13)

a) $T \sim$ Negative Binomial $(\theta, 1000)$

b) $\sqrt{I(\hat{\theta})}\,(\hat{\theta}-\theta) \sim N(0,1)$ where

here we have : $I(\theta) = \dfrac{1000(1-\theta)}{\theta^2(1-\theta)^2} \left(= -E\left(\dfrac{\partial^2}{\partial\theta^2} \ln f(t;\theta) \right) \right)$

$$\Rightarrow \mathbb{P}\left(-z_{1-\alpha/2} \le \sqrt{\frac{1000(1-\hat{\theta})}{\hat{\theta}^2(1-\hat{\theta})^2}} \cdot (\hat{\theta}-\theta) \le z_{1-\alpha/2} \right) \approx 1-\alpha$$

where $\hat{\theta} = \dfrac{1000}{\bar{X}}$

$\Rightarrow \alpha = .1$, $t = 5937$

$[0.1604, 0.1764]$ 90% CI for θ.