

HW # 6

11.2.4)

$$a) \quad b = (X^T X)^{-1} X^T Y = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$b) \quad e = \hat{y} - y = (-1 \ 0 \ 1 \ 0 \ -1)^T, \quad X^T e = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$d) \quad p(y|x_i) = c_i x_i; \quad c_i = \frac{X_i^T Y}{X_i^T X_i}; \quad i=1,2$$

$$\Rightarrow c_1 = 6; \quad c_2 = 14/3$$

$$\Rightarrow \hat{y} = 5x_1 + 3x_2 = 6x_1 + 14/3 x_2$$

11.2.6)

$$a) \quad b_2 = \frac{S_{xy}}{S_{xx}} \approx 9.71; \quad b_1 = \bar{y} - b_2 \bar{x} \approx -12856.38$$

$$b) \quad SST = S_{yy} = 7118267246$$

$$SSR = (S_{xy})^2 / S_{xx} = 3032350922$$

$$SSE = SST - SSR = 4085916324$$

$$c) \quad \hat{y}_i = 0.0038525 x_i^{1.8925}$$

11.3.1)

$$a) \quad 100\% \text{ CI for } \beta \text{ is: } \hat{\beta} \pm t_{n-1, \frac{\alpha+1}{2}} \cdot S_{\hat{\beta}}$$

$$\Rightarrow 95\% \text{ CI is: } [.591, 3.408]$$

$$b) \quad L(x_0) = .591x_0; \quad U(x_0) = 3.408x_0$$

11.3.3)

$$a) \quad S^2 = \frac{(Y - \hat{Y})^T (Y - \hat{Y})}{n-2} = 4/3$$

$$b) \quad \frac{\hat{\eta} - \eta}{S_{\hat{\eta}}} \sim t_{n-2} \quad \text{where } \hat{\eta} = C\hat{\beta}$$

$$\Rightarrow \hat{\eta} = 4; \quad S_{\hat{\eta}} = 2.18447$$

$$\Rightarrow 95\% \text{ CI for } \eta: \quad [-2.95, 10.95]$$

11.3.3)

$$c) \quad a = \begin{pmatrix} 1.237 \\ .868 \\ -1.105 \\ .132 \\ -.237 \end{pmatrix}; \quad a^T y = 4 = \hat{\eta}; \quad h = \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \Rightarrow E(h^T y) = \eta$$

$$\begin{aligned} \Rightarrow \text{Var}(\hat{\eta}^*) &= \|h\|^2 \sigma^2 = \|h-a+a\|^2 \sigma^2 \\ &= \|h-a\|^2 \sigma^2 + \|a\|^2 \sigma^2 \\ &= \|h-a\|^2 \sigma^2 + \text{Var}(\hat{\eta}) \end{aligned}$$

11.3.4)

$$a) \quad X = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}^T$$

$$Y = X\mu + \varepsilon; \quad \varepsilon_{ij} = Y_{ij} - \mu_i; \quad \varepsilon \sim N(0, \sigma^2 I_7)$$

$$b) \quad \hat{\mu}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij} \Rightarrow \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ 13 \end{pmatrix}$$

$$c) \quad S^2 = \frac{1}{n-3} \sum_{i=1}^3 \sum_{j=1}^{n_i} (y_{ij} - \hat{\mu}_i)^2 \Rightarrow S^2 = 4.5$$

$$d) \quad \frac{\hat{\eta}_{2,3} - \eta_{2,3}}{h_{2,3}} \sim t_{n-3} \quad \text{where: } h_{2,3}^2 = S^2 \left(\frac{1}{n_2} + \frac{1}{n_3} \right)$$

$$\Rightarrow 95\% \text{ CI for } \eta_{2,3}: [-8.376, 2.376]$$

$$95\% \text{ CI for } \eta_{1,3}: [-11.376, -.625]$$

$$e) \quad \hat{R} = \frac{\sum_{i=1}^3 n_i (\hat{\mu}_i - \hat{\mu})^2}{\sum_{i=1}^3 \sum_{j=1}^{n_i} (y_{ij} - \hat{\mu})^2} \Rightarrow \hat{R} = 2/3$$

$$\hat{\mu}_1 = \hat{\mu}_2 = \hat{\mu}_3 \Rightarrow \hat{R} = 0$$

$$(\alpha_{ij} = \alpha_{j,k} \quad \forall j \neq k, i=2,3) \Rightarrow \hat{R} = 1$$