STT 490-730  Exam 1  9-20-13  No notes/electronics.  
Closed book, 15 questions each 5 pts

I need your answers in terms of numbers and in some cases logs of numbers, not further evaluated or reduced to a number. Some problems ask you to ‘explain’ or write code.

1. Here are the prices of z-corp stock at the close of business:

<table>
<thead>
<tr>
<th>Day</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>393</td>
</tr>
<tr>
<td>Tuesday</td>
<td>398</td>
</tr>
</tbody>
</table>

Determine the price relative going from Monday to Tuesday. Do not reduce your answer.

\[
F_{\text{Tuesday}} = \frac{398}{393}
\]

2. I buy one share in a stock Monday at market opening. It is priced at $12. I keep the stock through 5 periods, selling just at the end of the fifth period. Here are the price relatives for each of those periods 1 through 5:

<table>
<thead>
<tr>
<th>Day</th>
<th>Price Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>.9</td>
</tr>
<tr>
<td>T</td>
<td>1.2</td>
</tr>
<tr>
<td>W</td>
<td>1.1</td>
</tr>
<tr>
<td>Th</td>
<td>.8</td>
</tr>
<tr>
<td>F</td>
<td>1.4</td>
</tr>
</tbody>
</table>

At the close of period 5 what is the PRICE of my one share? Express your answer in terms of numbers but do not reduce.
3-6. Here are price relatives for two investment A, B.

\[
\begin{array}{ccc}
X_1 & X_2 & \frac{X_n}{X_{n-1}} \\
A & 1.3 & 1.1 \\
B & 0.9 & 1.6 \\
\end{array}
\]

Give correct numerical expressions for questions 3-6 below. Do not simplify or reduce your answers.

3. Give the return from investing $1 in A through two periods.

\[\text{return of } \$1 \text{ in } A = 1 \cdot (1.3 \cdot 1.1)\]

4. Give the return from investing $1 in B through two periods.

\[\text{return of } \$1 \text{ in } B = 1 \cdot (0.9 \cdot 1.6)\]

5. Give the return from investing $0.5 on A, 0.5 on B and holding those two investment for two periods (this is a diversified buy and hold strategy).

\[\text{return} = 0.5 (1.3 \cdot 1.1) + 0.5 (0.9 \cdot 1.6)\]

6. Give the return from investing half of $1 on each of A and B, to begin with, then for the second round dividing your then existing fortune equally between A, B. This is a ‘re-balancing strategy’ since you are shifting investments between A and B. Transaction costs are assumed to be zero.

\[\text{Return period 1} = 0.5(1.3) + 0.5(0.9)\]

\[\text{Return period 2} = 0.5(0.5(1.3) + 0.5(0.9))\]

\[\text{Total return} = [0.5(1.3) + 0.5(0.9)] \cdot [0.5(0.5(1.3)) + 0.5(0.5(0.9))]\]
7. Horse race example. You can place a bet on a particular horse to win the race. Suppose the horse has probability 0.55 of winning that race and pays 2.1 : 1. What portion of $1 will Kelly bet on the horse winning? Use the Kelly short cut answer for horse race examples to express your answer in terms of 0.55 and 2.1 but do not reduce.

According to Kelly, if \( \frac{\frac{1}{p(\mathcal{H})}}{\frac{1}{\sigma_+}} \leq 1 \) we hold nothing back and bet winning probability. If the above is false, then use \( \frac{1-p(\mathcal{H})}{1-\sigma_+} \) to hold back; \( p(\mathcal{H}) = 0.3; \sigma_+ = 0.8 \); \( \sigma_+ = odds \ of \ winning \).

\[
1 - \frac{0.55}{1 - \frac{1}{2.1}} \Rightarrow \text{ambil. to hold back}
\]

Kelly will bet \( 1 - \left( \frac{1 - 0.55}{1 - \left( \frac{1}{2.1} \right)} \right) \approx 0.55 \).

8. Horse race example for large payout. You can place a bet on a particular horse to win a particular race. The horse has probability \( p = 0.1 \) of winning that race and pays \( M : 1 \). What portion of $1 will Kelly bet on the horse if \( M \) is very large? Take the limit as \( M \to \infty \) of Kelly's solution, expressing the answer as a number.

\[
\text{amb. to hold back} = \frac{1-p}{1-\sigma_+} = \frac{1-0.1}{1-\left( \frac{1}{M} \right)} \to \frac{0.9}{1-\left( \frac{1}{M} \right)} \quad \text{as} \quad M \to \infty
\]

\[
\lim_{M \to \infty} \frac{0.9}{1-\left( \frac{1}{M} \right)} = 0.9 = 1 - p
\]

9. In general, how does Kelly bet if payouts in a problem are very large?

Kelly bets the probability of winning \( (p = 0.1) \).

Your source is \( \frac{\frac{1}{\sigma_+}}{\frac{1}{p(\mathcal{H})}} \leq 1 \Rightarrow \frac{\frac{1}{\sigma_+}}{\frac{1}{M}} \) with \( \lim_{M \to \infty} \frac{1}{\sigma_+} = 0 \leq 1 \).
10. The typical gambler/investor will not place any money on an outcome for which
\[ p(\text{outcome}) (\text{return from dollar be on that outcome}) < 1. \]
Are there games/investments in which Kelly does put money on such outcomes? Give no example, just yes or no and cite a source.

Yes; Source: Kelly revision notes where they talk about if the game has some high payouts, but there are bets (that lose money) you must undertake in order to get the high payouts.

11. There are two stocks A, B. At each step with probability 1/2 one of the following two things happens:

- price relative of A is 1.4 and price relative for B is 0.6
- or price relative of A is 0.6 and price relative of B is 1.4.

In terms of the logs and numbers, give an expression for expected \( \log(\text{return on } $1) \) for a single play in which we divide $1 equally between A and B. Your answer must be in terms of numbers and log function, not reduced to a number.

\[
E \left( \ln(\text{return on }$1) \right) = 0.5 \ln \left( (0.5 \times 1.4) + (0.5 \times 0.6) \right) + 0.5 \ln \left( (0.5 \times 0.6) + (0.5 \times 1.4) \right)
\]

12. How would the numerical answer to #11 affect Kelly's decision as to whether money will be made in the long run?

Since we are splitting our $1 equally, Kelly would decide that the best we could do is double our money (go nowhere).
13. Write code for an R function that accepts a probability vector ‘p’ on possible returns r (also a vector) on a dollar and calculates $E \log(\text{return on a dollar}) := \sum p(i) \log(r(i))$.

```r
fortune <- function(p, r) { 
  return(sum(p * log(r)) / sum(p))
}
```

14. Write R code for plotting the trajectory of values of a vector v as a function of indices 1 to length[v]. It is simple code.

```r
plot(v, 1:length(v), main = "Trajectory of values")
```

15. Write R code that multiplies two matrices A, B to form matrix product $AB$ if the number of columns of A is equal to the number of rows of B.

```r
A <- matrix(c(4, 5, 6), nrow = 3, ncol = 2) # Example matrix A
B <- matrix(c(2, 3, 4), nrow = 2, ncol = 3) # Example matrix B

matmult <- A %*% B
matmult
```
**Bonus.** Explain what these pictures are illustrating.

These pictures are illustrating fortune of Kelly's principle versus fortune of a learning algorithm. In the top two graphs, the learning algorithm is behind the Kelly algorithm, but is on the same path as Kelly. The third chart shows the learning algorithm beating Kelly in first twenty cycles. However in the fourth chart, Kelly takes off and leaves the learning algorithm behind (at 100 cycles).