5 + 9 = 14 - 2
6 = 2 + 3 \times 5 - 9
1a. Fill in the boxes consistent with

\[
\begin{align*}
P(D) &= 0.2 \\
P(+|D) &= 0.9 \\
P(-|D^c) &= 0.8
\end{align*}
\]

1b. \( P(D|\neg) \) =

\[
\frac{0.2 \times 0.1}{0.2 \times 0.1 + (1 - 0.1) \times 0.8} = \frac{0.02}{0.02 + 0.72} = \frac{1}{37} \\
(1 - 0.2)(0.1) = 0.08
\]

2. A

Component failures are independent. Failure probabilities are shown.

2a. Determine \( P(\text{at least 1 of A, B fails}) \)

\[
1 - (1 - 0.3) \times (1 - 0.4) = 0.79
\]

2b. \( P(\text{system fails to connect A with B}) \)

\[
1 - [(1 - 0.1)(0.2)] \times (1 - 0.3) \times (1 - 0.4) = 0.75
\]

3a. \( P(R > G + 1) \) Express as \( \frac{10}{36} \)

3b. \( P(R^2 < G) \)

Express as \( \frac{7}{36} \)

3c. \( P(R > G + 1) \cap (R^2 < G) \)

Express as \( \frac{9}{36} \)
4. \[ x \quad 3 \quad -1 \quad 1 \]
\[ p(x) \quad 0.1 \quad 0.4 \quad 0.5 \]

4a. \[ E(X^2) \text{ (sum over three cases above)} \]
\[ \begin{align*}
(0.1)^2 + (0.4)(-1)^2 + (0.5)(1)^2 &= 0.01 + 0.4 + 0.5 = 1.8
\end{align*} \]

4b. **Determine probability distribution of \( Y = X^2 \), from it calculate \( E(Y) \).**

\[ Y \quad 1 \quad 9 \]
\[ p(y) \quad 0.1 \quad (0.5) \]

\[ E(Y) = (0.1)1 + (0.9)9 = 1.8 \]

4c. \[ E\left(\frac{X}{X+3}\right) \text{ (sum over three cases for } X) \]
\[ = 0.1\left(\frac{3}{3+3}\right) + 0.4\left(\frac{-1}{-1+3}\right) + 0.5\left(\frac{1}{1+3}\right) = 0.1(0.5) + 0.4(-0.5) + 0.5(0.25) = 0.05 - 0.2 + 0.125 = -0.025 \]

4d. **\( \text{VAR}(X) \text{ from (4a) and } E(X) \text{ (short form)} \).**

\[ E(X) = 3(0.1) + 1(0.4) + 1(0.5) = 0.3 + 1.4 + 0.5 = 2.2 \]

\[ \text{VAR}(X) = E(X^2) - (E(X))^2 = 1.8 - (2.2)^2 = 1.8 - 4.84 = 0.2 \]

4e. \[ \text{VAR}(9X - 6) \text{ in terms of } \text{VAR}(X) \]
\[ \text{VAR}(9X - 6) = E[(9X - 6)^2] - [E(9X - 6)]^2 = 9[E(X^2) - 12X + 36] - [E(9X - 6)]^2 = 81 \cdot [E(X^2) - 12Ex + 36] - (9E(X) - 6)^2 = 81 \cdot [E(X^2) - (E(X))^2] \]

\[ \text{VAR}(9X - 6) = 81 \cdot \text{VAR}(X) \]

4f. \[ E(7X - 2X^2 - 1) \]
\[ 7E(X) - 2E(X^2) - 1 = 7(2.2) - 2(1.8) - 1 = 15.4 - 3.6 - 1 = 0.2 \]

4g. **Graph cumulative distribution function of \( X \).**

\[ F(Y) \]

\[ Y \]

---

4c. \[ E\left(\frac{X}{X+3}\right) \text{ (sum over three cases for } X) \]
\[ = 0.1\left(\frac{3}{3+3}\right) + 0.4\left(\frac{-1}{-1+3}\right) + 0.5\left(\frac{1}{1+3}\right) = 0.1(0.5) + 0.4(-0.5) + 0.5(0.25) = 0.05 - 0.2 + 0.125 = -0.025 \]

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\[ E(X) = 3(0.1) + 1(0.4) + 1(0.5) = 0.3 + 1.4 + 0.5 = 2.2 \]

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\[ \text{VAR}(9X - 6) = E[(9X - 6)^2] - [E(9X - 6)]^2 = 9[E(X^2) - 12X + 36] - [E(9X - 6)]^2 = 81 \cdot [E(X^2) - 12Ex + 36] - (9E(X) - 6)^2 = 81 \cdot [E(X^2) - (E(X))^2] \]

\[ \text{VAR}(9X - 6) = 81 \cdot \text{VAR}(X) \]

4f. \[ E(7X - 2X^2 - 1) \]
\[ 7E(X) - 2E(X^2) - 1 = 7(2.2) - 2(1.8) - 1 = 15.4 - 3.6 - 1 = 0.2 \]

4g. **Graph cumulative distribution function of \( X \).**
5. **3R 6G** Draws without replacement and with equal probabilities on those left.

5a. \( P(R_3 \mid G_5) \) use and state basic fact making it easy. Order doesn’t matter.

\[
P(R_3 \mid G_5) = \frac{3}{8} \times \frac{2}{8} = \frac{3}{32} \]

5b. \( P(R_3 \lor G_5) \) use and state principles/rules employed.

\[
P(R_3 \lor G_5) = P(R_3) + P(G_5) - P(R_3 \land G_5)
= \frac{3}{9} + \frac{2}{9} - \frac{1}{9} \frac{2}{9}
= \frac{3}{9} + \frac{1}{9} \frac{2}{9}
= \frac{2}{9}
\]

5c. State the now obvious \( P(G_2) = \)

Prove it using probability rules:

\[
P(A) = \frac{\# \text{ desired outcomes}}{\# \text{ possible outcomes}} = \frac{6}{3+6} = \frac{6}{9}
\]

6a. Express \((38\choose3)\) numerically.

\[
(38\choose3) = \frac{38!}{3! \cdot 35!} = \frac{38 \cdot 37 \cdot 36}{3 \cdot 2 \cdot 1} = 38 \cdot 37 \cdot 6
\]

6b. Give the definition of \((38\choose3)\) (not formula) "38 choose 3". The number of combinations of 3 items picked uniquely from 38 items.

6c. Give the idea of why \((38\choose3)\) involves factorials. That is, the proof of the formula (A)

\[
\begin{align*}
\# \text{ of permutations} &= 38 \cdot 37 \cdot 36 = \frac{38!}{35!} \\
\# \text{ permutations with repeated letters} &= \frac{38!}{3!} \\
\# \text{ repeated permutations} &= 3 \cdot 2 \cdot 1 = 3 \\
\end{align*}
\]

(because 3 were picked)

7. Population AA of N individuals. 2 a. 1. 7.

\[
P = \frac{\# A \text{ letters in population}}{2N} = \frac{2 \times 0.25 \times 0.7}{2 \times 0.41} = 0.25
\]

7b. \( P(\text{offspring} : AA) \) random mating.

\[
P^2 = (0.25)^2 = 0.0625
\]

\(\neg\text{Reduce} \)