Class Notes 9 4 15

Referring to the three sector wheel, each sector having the same probability 1/3 of stopping opposite the pointer when spun, with respective payouts 2x, 4x, 8x for money bet on the individual sectors.

In the plot below we compare the growth of (natural) log of fortune earned by each of two betting schemes, applied over 200 fresh spins of the wheel.

(1) Hold back nothing, betting 1/3 of current fortune on each of 2x, 4x, 8x.
(2) Hold back nothing, betting .5, .3, .2 of current fortune on each of 2x, 4x, 8x.

Plot (1), shown thickest, earns most. Theoretically, the probability average return from (1) over 200 plays should be 200 (1/3) (log(2/3) + log(4/3) + log(8/3)) \sim 57.53641.

Plot (2), shown thinner, earns less. Theoretically, the probability average return from (2) over 200 plays should be around 200 (1/3) (log(2 0.5) + log(4 0.3) + log(8 0.2)) \sim 43.48835.

Keep in mind that the actual fortune is e to the power of the log_e fortune shown. That is a big deal and can be a scary ride.

Here is R code to calculate the anticipated log fortune (1) after 200 wheel spins.

\[
> 200*(1/3)*\text{sum}(\log(c(2,4,8)*c(1/3,1/3,1/3)))
\]

[1] 57.53641
Here is the same calculation for (2).

> 200*(1/3)*sum(log(c(2,4,8)*c(.5,.3,.2)))
[1] 43.48835

Here is R code for the simulation and thick plot (1).

```r
fortunes=rep(1,n)
wheel=function(bets,n){
  hold=1-sum(bets)
  pay=hold+c(2,4,8)*bets
  for(i in 1:(n-1)){
    fortunes[i+1]=fortunes[i]*(sample(pay,1))
    plot(log(fortunes))
  }
}

> wheel(c(1/3,1/3,1/3), 200)
```

Here is R code for the simulation and thin plot (2). R command “plot” was replaced with “lines” in order to overlay the two plots (“lines” is another R coding term).

```r
fortunes=rep(1,n)
wheel2=function(bets,n){
  hold=1-sum(bets)
  pay=hold+c(2,4,8)*bets
  for(i in 1:(n-1)){
    fortunes[i+1]=fortunes[i]*(sample(pay,1))
    lines(log(fortunes))
  }
}

> wheel2(c(.5,.3,.2), 200)
```

**Thanks to your classmate** who suggested (2) which cannot lose money and holds nothing back. It is less volatile than (1) which “maximizes the average log return per dollar invested.”

The latter is called the Kelly Principle or Kelly-Breiman Principle. A popular account by Poundstone titled “Fortune’s Formula” explains some of its applications and history.

*If you think like Kelly you eye each investment opportunity through the lens of maximizing the probability-average, of the logarithm of, the relative rate of return per dollar invested.*