1. Suppose that $X$ is a discrete random variable with
   $P[X = 0] = \frac{1}{3}, P[X = 1] = \frac{2\theta}{3}, P[X = 2] = \frac{1 - \theta}{2}$ and $P[X = 3] = \frac{1 - \theta}{2}$.
   Five independent observations of $X$ are made:
   $x_1 = 1, x_2 = 0, x_3 = 3, x_4 = 2, x_5 = 0$.
   Find the maximum likelihood estimate of $\theta$.

2. Suppose that the data $(X_1, \ldots, X_n)$ are i.i.d. with
   $X$. The density function of $X$ is
   $f(x|\theta) = e^{-x - 2\theta}, x \geq 2\theta$ and $f(x|\theta) = 0$, otherwise.
   Find the maximum likelihood estimator of $\theta$.

3. Let $X_1, \ldots, X_n$ i.i.d. $N(\mu, \sigma^2)$, where $N(\mu, \sigma^2)$
   denotes the normal distribution with mean $\mu$
   and Variance $= \sigma^2$. Derive the Cramér–Rao
   lower bound of $\sigma^2$.

   From text book:
   
   Section 8.10
   
   (Question numbers: 50, 51, 52, 73)