Homework for 1/13 Due 1/22

1. [§5-23] An irregularly shaped object of unknown area $A$ is located in the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$. Consider a random point distributed uniformly over the square; let $Z = 1$ if the point lies inside the object and $Z = 0$ otherwise. Show that $E[Z] = A$. How could $A$ be estimated from a sequence of $n$ independent points uniformly distributed on the square?

(*Hint:* Imagine this is actually a coin tossing experiment with unknown probability of getting Head, that is, the coin land on H if the point is inside the object and on T otherwise. How will you estimate the probability of getting H?)

2. [§5-26] Suppose that a basketball player can score on a particular shot with probability .3. Use the central limit theorem to find the approximate distribution of $S$, the number of successes out of 25 independent shots. Find the approximate probabilities that $S$ is less than or equal to 5, 7, 9, and 11 and compare these to the exact probabilities. (*Hint:* Let $X_1, X_2, \ldots, X_{25}$ be the indicator random variables of the 25 shots, that is, $X_i = 1$ if the player scores on the $i$th shot and $X_i = 0$ otherwise.)