Homework for 1/8

1. [§4-6] Let $X$ be a continuous random variable with probability density function $f(x) = 2x, 0 \leq x \leq 1$.

   (a) Find $E[X]$.
   (b) Find $E[X^2]$.
   (c) Find $\text{Var}[X]$.

3. [§4-42] Let $X$ be an exponential random variable with standard deviation $\sigma$. Find $\Pr(|X - \mathbb{E}[X]| > k\sigma)$ for $k = 2, 3, 4$, and compare the results to the bounds from Chebyshev’s inequality.

4. [§4-80] Let $X$ be a continuous random variable with density function $f(x) = 2x$, $0 \leq x \leq 1$. Find the moment-generating function of $X$, $M(t)$, and verify that $\mathbb{E}[X] = M'(0)$ and that $\mathbb{E}[X^2] = M''(0)$. 
5. [§4-91] Use the mgf to show that if $X$ follows an exponential distribution, 
$cX (c > 0)$ does also. (*Hint:* find the mgf of $cX$ and see what this mgf looks like.)