Case 1: Smith, a 40 year old man, takes out a 30-year mortgage that requires payments of 10000 at the end of each of the next 30 years. Smith also purchases a life insurance policy which will pay off the balance of the loan, if any, at the end of Smith’s death year. Expenses are zero. Suppose you are given

- \( i = 4\% \)
- \( a_{40:30} = 15.292 \).

Case 2: Consider a zero-expense, 20 year endowment policy purchased by a life aged 50. Level premiums of 23500 per year are payable annually throughout the term of the policy. A sum insured of 700000 is payable at the end of the term if the life survives to age 70. On death before age 70, a sum insured is payable at the end of the year of death equal to the policy value at the start of the year in which the policyholder dies. Assume

- \( i = 3.5\% \) per year
- \( q_x = 0.05 \) for every \( x \in \{65, 66, 67, 68, 69, 70\} \).

Case 3: A whole-life policy (with no upper bound \( \omega \) on maximum age) is valued continuously, with

\[
\begin{align*}
S_t &= 2_t V \\
\varepsilon_t &= 0 = E_t \\
\delta_t &= \mu_{[x] + t} = \lambda > 0 \\
P_t &= P > 0.
\end{align*}
\]
1. For Case 1, under the EPP, for the single benefit premium $P$,

(a) $P = 5000$
(b) $P = 7500$
(c) $P = 10000$
(d) $P = 20000$
(e) $P$ cannot be computed with the information given.

ANSWER: d.)

$$
0 = 0 = 0 \quad V = \mathbb{E}[PV(Liability)] = \mathbb{E}[PV(Loan)] - \mathbb{E}[PV(\text{Smith’s Payments})] - P \\
= 10000(\ddot{a}_{30}^{(0.04)}) - 10000a_{40.04} - P \\
= 10000 \frac{1 - (1.04)^{-30}}{0.04} - 152920 - P \\
= 172920 - 152920 - P = 20000 - P. \tag{2}
$$

2. For Case 1, before solving for $P$, the policy value at time 1 can be written as

(a) $\dot{V} = 1.04P + q_{[40]}(\ddot{a}_{30}^{(0.04)}) + \dot{V}$
(b) $\dot{V} = 1.04P + q_{[40]}(\dot{V})$
(c) $\dot{V} = 1.04P - (\ddot{a}_{30}^{(0.04)}) q_{[40]}$
(d) $\dot{V} = 1.04P + (\ddot{a}_{30}^{(0.04)}) q_{[40]}$
(e) None of the above.

ANSWER: e.) Under the recursive formula,

$$
0 = \dot{V} = -P + q_{[40]} \frac{10000(\ddot{a}_{30}^{(0.04)})}{1.04} + p_{[40]} \dot{V} \frac{1.04}{1.04} \\
\Rightarrow \dot{V} = \frac{1.04P - 10000q_{[40]}(\ddot{a}_{30}^{(0.04)})}{P_{[40]}} \tag{3}
$$
3. For Case 2, the policy value at time 19 is

(a) \(19V = 650434\)
(b) \(19V = 602629\)
(c) \(19V = 600000\)
(d) \(19V = 597432\)
(e) None of the above.

4. For Case 2, the policy value at time 18 is

(a) \(18V = 650434\)
(b) \(18V = 602629\)
(c) \(18V = 600000\)
(d) \(18V = 597432\)
(e) None of the above.

ANSWERS: 3a.) and 4b.) It follows that

\[
S_{t+1} = tV \\
e_t = 0 = E_t \\
S = 700000 \\
P_t = 23500
\]

\[
tV = -23500 + q_{50+t} \frac{tV}{1.035} + p_{50+t} \frac{t+1V}{1.035}
\]

and so

\[
tV = \frac{p_{50+t} \cdot t+1V - 24322.50}{p_{50+t} + 0.035}
\]

\[
20 - V = 700000 \\
\Rightarrow \quad 19V = \frac{p_{69} \cdot (20 - V) - 24322.50}{p_{69} + 0.035}
\]

\[
= \frac{0.95 \cdot (700000) - 24322.50}{0.95 + 0.035} = 650434
\]

\[
18V = \frac{p_{68} \cdot (19V) - 24322.50}{p_{68} + 0.035}
\]

\[
= \frac{0.95 \cdot (650434) - 24322.50}{0.95 + 0.035} = 602629.
\]
5. For Case 2, if all the terms of the contract remain unchanged, except that now, $S_{t+1} = t_{+1}V$, calculate $19V$

(a) $19V = 700000$
(b) $19V = 652828$
(c) $19V = 650434$
(d) $19V = 602629$
(e) None of the above

ANSWER: b.)

By recursion,

$$S_{t+1} = t_{+1}V$$
$$e_t = 0 = E_t$$
$$S = 700000$$
$$P_t = 23500$$

$$tV = -23500 + q_{[50]+t} \frac{t_{+1}V}{1.035} + p_{[50]+t} \frac{t_{+1}V}{1.035}$$

$$= -23500 + \frac{t_{+1}V}{1.035}$$

$$\therefore 19V = -23500 + \frac{700000}{1.035}$$

$$= 652828.50.$$

Note that this is a higher value than obtained from the previous assumption that $S_{t+1} = tV$. 
6. For Case 3, if $P$ is determined such that $\theta V > 0$, is there a value $T > 0$ where $\theta V = 0$?

(a) Yes.
(b) No.
(c) Maybe, but need more information.

ANSWER: b.)

By the ode used for continuous valuation, we need only see that

$$\frac{d}{dt} (tV) = \delta_t \cdot tV + P_t - \epsilon_t - (S_t + E_t - tV) \mu_{[x]} + t$$

where $T > 0$

$$= \lambda \cdot tV + P - (2tV + 0 - tV) \mu_{[x]} + t$$

$$= P > 0$$

(7)

to conclude that the policy value will only continue to rise.

7. For Case 3, if the terms of the contract are maintained, except that now $S_t = 1000$, then the initial policy value is

(a) $\theta V = 500 - \frac{P}{2\lambda}$
(b) $\theta V = 500 + \frac{P}{2\lambda}$
(c) $\theta V = 1000e^{-\lambda} - P$
(d) None of the above.

ANSWER: a.)

$$tV = \int_0^\infty 1000e^{-\lambda s} \cdot \lambda e^{-\lambda s} ds - \int_0^\infty e^{-\lambda s} \cdot Pe^{-\lambda s} ds$$

$$= \frac{1000\lambda}{\lambda + \lambda} - \frac{P}{\lambda + \lambda} = 500 - \frac{P}{2\lambda}.$$