STT 459: Construction & Evaluation of Actuarial Models

Ashoke Kumar Sinha
Frequency and severity with coverage modifications

- Frequency and Severity
- Various type of coverage modifications
- Policies with deductible

Ordinary deductible
Franchise deductible
Policy Limits
Coinsurance
Deductibles and Claim Frequency

Frequency and severity with coverage modifications
In actuary we are mostly interested in

- **Frequency**: how frequently loss event occurs,
- **Severity**: the size of the loss events.

Since both of the above are non-deterministic and non-negative in nature, we model them using non-negative random variables.
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Frequencies are modeled using suitable non-negative discrete rv’s, and the corresponding probability distributions are known in actuarial literature as *frequency distributions*.

The severities, based on the type of coverage, are modeled using either continuous or mixed rv’s, and the corresponding probability distributions are known as *severity distributions*. 
Various types of coverage modifications

Insurance companies sell various types of policies. Here we shall discuss properties of the following:

- policies with deductible
  - ordinary deductible,
  - franchise deductible,
Various type of coverage modifications

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We shall also consider the effect of inflation on payment. Finally the effect of deductibles on claim frequency will be discussed.
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Relevant chapters from the textbook: Ch 8 and 3.
Policies with deductible

Suppose,

- \( X \): severity (a continuous non-negative rv with cdf \( F_X \)),
- \( d \): deductible amount where \( P(X > d) > 0 \).

In ordinary deductible case, the insurance company will pay nothing for any damage up to amount \( d \), and will pay \( (X - d) \) if the damage is more than \( d \).
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In **franchise deductible** case, the insurance company will pay nothing for any damage up to amount $d$, and will pay $X$ if the damage is more than $d$. 
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In both deductible situations, we need to distinguish between payment per-loss \((Y^L)\) and payment per-payment \((Y^P)\).
Policies with deductible

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In both deductible situations, we need to distinguish between payment per-loss ($Y^L$) and payment per-payment ($Y^P$).

N.B.: Since severity and payment are all non-negative in values, we shall consider their cdf, pdf, survival function etc. only for non-negative values.
Ordinary deductible

- $Y^L$ in ordinary deductible
- Moments of $Y^L$
- Loss elimination ratio
- Payment per-payment variable
- cdf, pdf, survival and hazard functions of $Y^P$
- Moments of $Y^P$
- Effect of inflation

Franchise deductible

Policy Limits

Coinsurance

Deductibles and Claim Frequency
**Payment per-loss \((Y^L)\) in ordinary deductible**

In ordinary deductible, the **payment per-loss** variable is defined as

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Y^L := (X - d)_+ = \begin{cases} 
0, & \text{if } X \leq d, \\
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**cdf:**

\[
F_{Y^L}(y) = \begin{cases} 
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\[ \text{Survival function:} \]

$$S_{Y^L}(y) = \begin{cases} S_X(d), & y = 0, \\ S_X(y + d), & y > 0. \end{cases}$$
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Note that at 0 there is a probability mass of \(F_X(d)\).
Moments of $Y^L$

The $k^{th}$ raw moment of $Y^L$ is

$$\mathbb{E}[(Y^L)^k] = \mathbb{E}[(X - d)^k_+] = \int_d^\infty (x - d)^k f_X(x) \, dx.$$
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Notice that

$$X = (X - d)_+ + (X \wedge d), \text{ where } X \wedge d = \min\{X, d\}.$$
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Notice that

$$X = (X - d)_+ + (X \wedge d), \quad \text{where} \quad X \wedge d = \min\{X, d\}.$$ 

$$\Rightarrow \quad \mathbb{E}(Y^L) = \mathbb{E}[(X - d)_+] = \mathbb{E}(X) - \mathbb{E}(X \wedge d).$$
The **loss elimination ratio** is the ratio of the decrease in the expected payment with an ordinary deductible to the expected payment without the deductible.

\[
\text{Loss elimination ratio} = \frac{\mathbb{E}(X) - \mathbb{E}[(X - d)_+]}{\mathbb{E}(X)} = \frac{\mathbb{E}(X \land d)}{\mathbb{E}(X)}.
\]

Loss elimination ratio is indicative of the percentage decrease in expected cost by introducing deductible, with respect to per unit expected cost without deductible.
In ordinary deductible, the **payment per-payment** variable is defined as

\[ Y^P := \begin{cases} \text{undefined,} & \text{if } X \leq d, \\ X - d, & \text{if } X > d. \end{cases} \]

So we do not observe \( Y^P \) if \( X \leq d \). However, if \( X > d \), then a payment of \( X - d \) is made.

Thus \( Y^P \) represents the amount paid given the information that a payment is made.
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Thus \( Y^P \) represents the amount paid given the information that a payment is made.

On the other hand, \( Y^L \) is the amount paid per-loss, which is 0 if \( X \leq d \), and \((X - d)\) if \( X > d \).

Notice, \( Y^P = Y^L | Y^L > 0 \).
For $y > 0$, the cdf:

$$F_{YP}(y) = \frac{F_X(y + d) - F_X(d)}{1 - F_X(d)} = \frac{F_X(y + d) - F_X(d)}{S_X(d)},$$
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and survival function:

$$S_{YP}(y) = \frac{1 - F_X(y + d)}{1 - F_X(d)} = \frac{S_X(y + d)}{S_X(d)}.$$

The pdf:

$$f_{YP}(y) = \frac{f_X(y + d)}{S_X(d)}.$$
For \( y > 0 \), the cdf:

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F_{Y^P}(y) = \frac{F_X(y + d) - F_X(d)}{1 - F_X(d)} = \frac{F_X(y + d) - F_X(d)}{S_X(d)},
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The pdf:

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f_{Y^P}(y) = \frac{f_X(y + d)}{S_X(d)}.
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Hazard function:

\[
h_{Y^P}(y) = h_X(y + d) = h_{Y^L}(y).
\]
Moments of $Y^P$

The $k^{th}$ raw moment of $Y^P$ is

$$
\mathbb{E}[(Y^P)^k] = \frac{\int_{d}^{\infty} (x - d)^k f_X(x) \, dx}{S_X(d)}.
$$
Moments of $Y^P$

The $k^{th}$ raw moment of $Y^P$ is

$$E[(Y^P)^k] = \frac{\int_0^{\infty} (x - d)^k f_X(x) dx}{S_X(d)}.$$  

In particular, we define mean excess loss function

$$e_X(d) = E(Y^P) = \frac{\int_0^{\infty} S_X(x) dx}{S_X(d)} = \frac{\int_0^{\infty} S_X(x + d) dx}{S_X(d)},$$
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Note:

$$
e_X(d) = \frac{\mathbb{E}(Y^L)}{S_X(d)} = \frac{\mathbb{E}(X) - \mathbb{E}(X \wedge d)}{S_X(d)}.
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Moments of $Y^P$

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Note:

- $e_X(d) = \frac{E(Y^L)}{S_X(d)} = \frac{E(X) - E(X \land d)}{S_X(d)}.$
- If $P(X > 0) = 1$, i.e. $X$ is positive almost surely, then $e_X(0) = E(X).$
Effect of inflation

Consider an ordinary deductible of $d$, and a uniform inflation $1 + r$.

Define, $d^* = \frac{d}{1 + r}$.

The effect of inflation on $\mathbb{E}(Y^L)$ is:

$$(1 + r)\{\mathbb{E}(X) - \mathbb{E}(X \wedge d^*)\} = (1 + r)\mathbb{E}[(X - d^*)_+] .$$
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If $F_X(d^*) < 1$, the effect of inflation on $\mathbb{E}(Y^P)$ is:

$$\frac{(1 + r)\{\mathbb{E}(X) - \mathbb{E}(X \wedge d^*)\}}{S_X(d^*)} = \frac{(1 + r)\mathbb{E}[(X - d^*)_+]}{S_X(d^*)}.$$
Franchise deductible

Frequency and severity with coverage modifications

Ordinary deductible

Franchise deductible

- $Y^L$ in franchise deductible
- $\mathbb{E}(Y^L)$
- Payment per-payment variable
- $\mathbb{E}(Y^P)$

Policy Limits

Coinsurance

Deductibles and Claim Frequency

Franchise deductible
Payment per-loss ($Y^L$) in franchise deductible

In franchise deductible, the payment per-loss variable is defined as

$$Y^L := \begin{cases} 0, & \text{if } X \leq d, \\ X, & \text{if } X > d. \end{cases}$$
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Note that at 0 there is a probability mass of \(F_X(d)\).
\[ \mathbb{E}(Y^L) = \int_d^\infty x f_X(x) \, dx \]
\[ = \int_d^\infty S_X(x) \, dx + dS_X(d) \]
\[ = \int_0^\infty S_X(x + d) \, dx + dS_X(d). \]

Notice that, \( \int_d^\infty S_X(x) \, dx \) is the expected cost per-loss in ordinary deductible, which is also equal to
\[ \mathbb{E}(X) - \mathbb{E}(X \wedge d). \]

Hence,
\[ \mathbb{E}(Y^L) = \mathbb{E}(X) - \mathbb{E}(X \wedge d) + dS_X(d). \]
In franchise deductible, the **payment per-payment** variable is defined as

\[
Y^P := \begin{cases} 
\text{undefined,} & \text{if } X \leq d, \\
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i.e. \( Y^P = Y^L | Y^L > 0 \).
### Payment per-payment variable

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\[ \mathbb{E}(Y^P) = \frac{\int_{d}^{\infty} x f_X(x) \, dx}{S_X(d)} \]

\[ = \frac{\int_{d}^{\infty} S_X(x) \, dx}{S_X(d)} + d \]

\[ = \frac{\int_{0}^{\infty} S_X(x + d) \, dx}{S_X(d)} + d. \]
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Since \( E(Y^P) = \frac{\mathbb{E}(Y^L)}{S_X(d)} \), an alternative expression is:

\[ \mathbb{E}(Y^P) = \frac{\mathbb{E}(X) - \mathbb{E}(X \wedge d)}{S_X(d)} + d. \]
Policy Limits

Frequency and severity with coverage modifications

Ordinary deductible

Franchise deductible

Policy Limits
- Policy limits

Coinsurance

Deductibles and claim frequency
In a **policy limit** coverage with limit $u$, the insurance pays the full amount if the loss is below $u$, but for losses above $u$ the insurance only pays $u$. Thus in policy limit the payment (per-loss) is: $Y = X \wedge u = \min\{X, u\}$. 
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CDF:

$F_Y(y) = \begin{cases} F_X(y), & \text{if } y < u, \\ 1, & \text{if } y \geq u. \end{cases}$

PDF:

$f_Y(y) = \begin{cases} f_X(y), & y < u, \\ 1 - F_X(u), & y = u. \end{cases}$


# Policy limits

In a **policy limit** coverage with limit \( u \), the insurance pays the full amount if the loss is below \( u \), but for losses above \( u \) the insurance only pays \( u \). Thus in policy limit the payment (per-loss) is: 

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Y = X \wedge u = \min\{X, u\}.
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## CDF

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\mathbb{E}[(X \wedge u)^k] = \int_0^u x^k f(x)dx + u^k[1 - F_X(u)].
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**Policy limits**

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\[
\mathbb{E}[(X \land u)^k] = \int_0^u x^k f(x)dx + u^k[1 - F_X(u)].
\]

After uniform inflation of \( 1 + r \), the expected cost is

\[
(1 + r)\mathbb{E}(X \land u^*) = \frac{u}{1 + r}.
\]
Frequency and severity with coverage modifications

Ordinary deductible

Franchise deductible

Policy Limits

**Coinsurance**
- Coinsurance with deductibles and limits
- Expected costs

Deductibles and Claim Frequency
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$$Y^L = \begin{cases} 
0, & \text{if } X < d^*, \\
\alpha[(1 + r)X - d], & \text{if } d^* \leq X < u^*, \\
\alpha(u - d), & \text{if } X \geq u^*,
\end{cases}$$

where $d^* = d/(1 + r)$, and $u^* = u/(1 + r)$. 
Expected costs

In this case,

\[ E(Y^L) = \alpha (1 + r) [E(X \land u^*) - E(X \land d^*)]. \]
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If \( F_X(d^*) < 1 \), then

\[ \mathbb{E}(Y^P) = \frac{\mathbb{E}(Y^L)}{S_X(d^*)}, \quad \text{and} \quad \mathbb{E}[(Y^P)^2] = \frac{\mathbb{E}[(Y^L)^2]}{S_X(d^*)}. \]
<table>
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<th>Frequency and severity with coverage modifications</th>
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<td>Coinurance</td>
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**Deductibles and Claim Frequency**

- Impact of deductibles on claim frequency
Impact of deductibles on claim frequency

Impact of deductibles on claim frequency

To be written later.