\( X_1, \ldots, X_n \sim E(\theta, 1), \quad \theta \in \mathbb{R} \)

\( p(x) = \exp \left\{ -\sum_{i=1}^n (x_i - \theta)^2 I(x_i > \theta) \right\} = I(x_0 > \theta) e^{-\frac{x_0}{\theta}} + \theta \)

By Factorization Th., \( T = X_0 \) is sufficient.

Consider \( \frac{p(T)}{p_0} = \frac{I(x_0 > \theta_0)}{I(x_0 > \theta)} e^{\theta(\theta_1 - \theta_0)}. \) It is a f-n of \( T \Rightarrow T \) is minimal.

(b) \( E(0, b), \quad b > 0 \)

\( p(x) = \frac{1}{b^n} \exp \left\{ -\frac{\sum_{i=1}^n x_i}{b} \right\}, \quad x_i > 0 \)

By Factorization Th., \( T = \sum x_i \) is sufficient. This is exp. family of full rank \( \Rightarrow T \) is minimal.

(c) \( E(a, b), \quad a \in \mathbb{R}, \quad b > 0 \)

\( p(x) = \frac{1}{b^n} \exp \left\{ -\frac{\sum_{i=1}^n (x_i - a)}{b} \right\} I(x_0 > a) \)

By Factorization Th., \( T = (x_0, \sum_{i=1}^n (x_i - x_0)) \) is sufficient.

Consider \( \frac{p(T)}{p_0} = \frac{b_0^n}{b^n} \exp \left\{ -(\frac{1}{b_1} - \frac{1}{b_0}) \sum_{i=1}^n x_i + \frac{a_1}{b_1} - \frac{a_0}{b_0} \right\} \frac{I(x_0 > a_0)}{I(x_0 > a)} \)

It is a f-n of \( T \Rightarrow T \) is minimal.

6.18 \( X_1, \ldots, X_n \sim E(a, b) \)

\( P(X_0 \leq x) = 1 - P(X_0 > x) = 1 - b^n P(X > x) \)

\( P(x) = + b P(X > x)^{n-1} \cdot P(x) \)

\( P(X_i > x) = \int_0^{\infty} \frac{1}{b} \exp \left\{ -\frac{x}{b} (a - a) \right\} I(u > a) du = \int_0^{\infty} e^{-y} e^{y} = e^{-x/b \cdot a} \)

\( y = \frac{u-a}{b} \quad dy = dy_b \)

\( P(x) = \frac{1}{b^n} e^{-b (a-x)/b} I(x > a) \Rightarrow X_0 \sim \text{Exp}(a, b/n) \)

By Basu Th., \( X_0 \) and \( \sum_{i=1}^n (x_i - x_0) \) are independent since \( X_0 \) is CSS for \( a \)

and \( \sum_{i=1}^n (x_i - x_0) \) is ancillary for \( a \).

By means of mgf, we have \( M_{\sum X_i} (t) = M_X (t) = M_{\sum (X_i - x_0)} (t) = M_{X_0} (t) \)