(1) The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1,2,...,6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).

(a) What is the probability that both selected setups are for laptop computers?
(b) What is the probability that both selected setups are desktop machines?
(c) What is the probability that at least one selected setup is for a desktop computer?
(d) What is the probability that at least one computer of each type is chosen for setup?

(2) Show that if one event $A$ is contained in another event $B$ (i.e. $A$ is a subset of $B$), then $P(A) \leq P(B)$. [Hint: For such $A$ and $B$, $A$ and $B \cap A'$ are disjoint and $B = A \cup (B \cap A')$, as can be seen from a Venn diagram.] For general $A$ and $B$, what does this imply about the relationship among $P(A \cap B)$, $P(A)$ and $P(A \cup B)$?

(3) For any event $A$ and $B$ with $P(B) > 0$, show that $P(A|B) + P(A'|B) = 1$. 
(4) If \( P(B|A) > P(B) \), show that \( P(B'|A) < P(B') \).

(5) Show that for any three events \( A, B \) and \( C \) with \( P(C) > 0 \), \( P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C) \).

(6) An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let \( A \) be the event that the Asian project is successful and \( B \) be the event that the European project is successful. Suppose that \( A \) and \( B \) are independent event with \( P(A) = .4 \) and \( P(B) = .7 \).

(a) If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.

(b) What is the probability that at least one of the two projects will be successful?

(c) Given that at least one of the projects is successful, what is the probability that only the Asian project is successful?

(7) Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .10, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?