1. A fair coin is flipped independently.

(a) What is the probability that the first two outcomes are heads given the first toss is a head? [3pts]

Let \( A = \{ \text{the first toss is a head} \} \) and \( B = \{ \text{the second toss is a head} \} \).

Then
\[
P(A \cap B | A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B) = 0.5.
\]

(b) What is the probability that the first four outcomes are HHTH? [3pts]

Because of independence,
\[
P(\{HHTH\}) = P(\{H\})P(\{H\})P(\{T\})P(\{H\}) = 0.5^4 = 1/16.
\]

(c) What is the probability that two heads appear among the first four flips. [4pts]

Let \( A = \{ \text{two heads appear among the first four flips} \} \). The probability is
\[
P(A) = \binom{4}{2}0.5^20.5^2 = 3/8.
\]
2. Consider the experiment of throwing fair dice.

(a) What is the probability that at least two 6 in 5 throws of one die? [5pts]

Let \( A = \{ \text{no 6 appears in 5 throws of one die} \} \), \( B = \{ \text{one 6 appears in 5 throws of one die} \} \) and \( C = \{ \text{at least two 6 in 5 throws of one die} \} \). Then \( P(C) = 1 - P(A) - P(B) \). We note that

\[
P(A) = \left(\frac{5}{6}\right)^5 \quad \text{and} \quad P(B) = 5\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4.
\]

Thus, the probability \( P(C) = 1 - \left(\frac{5}{6}\right)^5 - \left(\frac{5}{6}\right)^5 = 0.196 \).

(b) What is the probability that at least two double 6 in 30 throws of two dice? [5pts]

Let \( A = \{ \text{no double 6 appears in 30 throws of two dice} \} \), \( B = \{ \text{one double 6 appears in 30 throws of two dice} \} \) and \( C = \{ \text{at least two double 6 in 30 throws of two dice} \} \). Then \( P(C) = 1 - P(A) - P(B) \). We note that

\[
P(A) = \left(\frac{35}{36}\right)^{30} \quad \text{and} \quad P(B) = 30\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{29}.
\]

Thus, the probability \( P(C) = 1 - \left(\frac{35}{36}\right)^{30} - 30\left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{29} = 0.202 \).
3. A total of 600 married working couples were polled about their annual salaries, with the following information resulting: For instance, in 136 of the couples, the wife earned more and the husband earned less than $25,000. If one of the couples is randomly chosen,

(a) what is the probability that the husband earns less than $25,000? [2pts]

From the table, we observed that the total number of husband that earns less than $25,000 is 212 + 136 = 348. Therefore, if a couple is randomly chosen, the probability that the husband earns less than $25,000 is

\[ P(\{\text{the husband earns less than $25,000}\}) = \frac{348}{600} = 0.58. \]

(b) what is the conditional probability that the wife earns more than $25,000 given that the husband earns more than $25,000? [4pts]

Let \( A = \{\text{the wife earns more than $25,000}\} \) and \( B = \{\text{the husband earns more than $25,000}\} \). The desired probability is

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{54}{198 + 54} = 0.214. \]

(c) what is the conditional probability that the husband earns more than $25,000 given that the wife earns less than $25,000? [4pts]

Let \( A = \{\text{the husband earns more than $25,000}\} \) and \( B = \{\text{the wife earns less than $25,000}\} \). The desired probability is

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{198}{198 + 212} = 0.483. \]