Homework 1 Due 1/21/15

1. [§4-6] Let $X$ be a continuous random variable with probability density function $f(x) = 2x, 0 \leq x \leq 1$.
   
   (a) Find $E[X]$.
   
   (b) Find $E[X^2]$.
   
   (c) Find $Var[X]$.

2. [§4-31] Let $X$ be uniformly distributed on the interval $[1, 2]$. Find $E[1/X]$.
   Is $E[1/X] = 1/E[X]$?

3. [§4-42] Let $X$ be an exponential random variable with standard deviation $\sigma$. Find $P(|X - E[X]| > k\sigma)$ for $k = 2, 3, 4$, and compare the results to the bounds from Chebyshev’s inequality.

4. [§4-80] Let $X$ be a continuous random variable with density function $f(x) = 2x, 0 \leq x \leq 1$. Find the moment-generating function of $X$, $M(t)$, and verify that $E[X] = M'(0)$ and that $E[X^2] = M''(0)$.

5. [§4-91] Use the mgf to show that if $X$ follows an exponential distribution, $cX (c > 0)$ does also.

6. [§5-16] Suppose that $X_1, \ldots, X_{20}$ are independent random variables with density functions
   
   $f(x) = 2x, 0 \leq x \leq 1$
   
   Let $S = X_1 + \cdots + X_{20}$. Use the central limit theorem to approximate $P(S \leq 10)$.

7. [§5-17] Suppose that a measurement has mean $\mu$ and variance $\sigma^2 = 25$.
   
   Let $X$ be the average of $n$ such independent measurements. How large should $n$ be so that $P(|X - \mu| < 1) = .95$?

8. [§5-18] Suppose that a company ships packages that are variable in weight, with an average weight of 15 lb and a standard deviation of 10. Assuming that the packages come from a large number of different customers so that it is reasonable to model their weights as independent random variables, find the probability that 100 packages will have a total weight exceeding 1700 lb.