Homework 2 Due 1/28/15

1. [§5-26] Suppose that a basketball player can score on a particular shot with probability .3. Use the central limit theorem to find the approximate distribution of $S$, the number of successes out of 25 independent shots. Find the approximate probabilities that $S$ is less than or equal to 5, 7, 9, and 11 and compare these to the exact probabilities. (*Hint: Let $X_1, X_2, \ldots, X_{25}$ be the indicator random variables of the 25 shots, that is, $X_i = 1$ if the player scores on the $i$th shot and $X_i = 0$ otherwise.*)

2. Assume $X \sim N(0, 1)$.
   (a) find $\mathbb{P}(X > 1.5)$;
   (b) find the value $c$ such that $\mathbb{P}(X < c) = 0.05$; and
   (c) find the value $d$ such that $\mathbb{P}(|X| < d) = 0.9$.

3. If $T \sim t_{20}$,
   (a) find $\mathbb{P}(T < -1.725)$;
   (b) find $\tau_1$ such that $\mathbb{P}(T > \tau_1) = .05$; and
   (c) find $\tau_2$ such that $\mathbb{P}(|T| < \tau_2) = .9$.

4. If $U \sim \chi^2_{6}$,
   (a) find $\mathbb{P}(U \leq 12.59)$; and
   (b) the value $c$ such that $\mathbb{P}(U > c) = 0.025$.

5. If $W \sim F_{2,3}$,
   (a) find $\mathbb{P}(W \geq 9.55)$; and
   (b) the value $c$ such that $\mathbb{P}(W > c) = 0.025$.

6. [§6-3] Let $\bar{X}$ be the average of a sample of 16 independent normal random variables with mean 0 and variance 1. Determine $c$ such that
   $\mathbb{P}(|\bar{X}| < c) = .5$.
   (*Review: if the pdf of a random variable $Y$ is symmetric (about its mean $\mu$), then $\mathbb{P}(Y - \mu < a) = \mathbb{P}(Y - \mu > -a)$.*
7. [§6-5] Show that if $X \sim F_{n,m}$, then $X^{-1} \sim F_{m,n}$.

8. [§6-10] Let $X_1, X_2, \ldots, X_{11}$ be i.i.d. random variables with $X_i \sim N(\mu, \sigma)$ for $1 \leq i \leq 11$, and $S^2$ be the sample variance. Use the chi-square distribution to calculate $\mathbb{P}(0.394 < S^2/\sigma^2 < 1.831)$. 