
INFO ON QUIZ 2.

Quiz 2 will be held on Monday, July 20. It will cover the following material:

3. Independence of events, pp.158-162. Exercise 3.21, Class 5 Notes, Class 6 Notes, Sample Quiz 2.
Probability of the Union.

**Problem 1.** 25% of the population in a given area is exposed to a television commercial for the Ford automobiles and 34% exposed to Ford’s radio advertisements. It is also known that 10% of the population is exposed to both means of advertising. If a person is randomly chosen out of the population of this area, what is the probability that he or she was exposed to at least one of the two modes of advertisement? (.49)

**Modified Problem 1.** In Problem 1 introduce the events $A= \{\text{The part of the population that is exposed to TV-advertisement}\}$ and $B= \{\text{The part of the population that is exposed to the radio advertisement}\}$. We still assume that $P(A)= .25$ and $P(B)= .34$. However let us assume that the events $A$ and $B$ are independent.
But we **no longer** assume that the intersection of A and B has a probability .10. Can we find the probability that he or she was exposed to at least one of the two modes of advertisement?

**Exercise 3.78.** Suppose

\( B_1, B_2, B_3 \) are mutually exclusive, and complementary

\[ P(B_1) = .2 \quad P(B_2) = .15 \quad P(B_3) = .65. \]

Consider another event A such that \( P(A \mid B_1) = .4, P(A \mid B_2) = .25 \)

\( P(A \mid B_3) = .6 \)

Find first \( P(A) \) and then

a. \( P(B_1 \mid A) \) \( (.158) \)

b. \( P(B_2 \mid A) \) \( (.074) \)

c. \( P(B_3 \mid A) \) \( (.768) \)
Test for Dawn Syndrome. Exercise 3.80, p.171.
Dawn Syndrome is caused by an abnormal gene of the fetus.

In 2007 two statisticians from San Diego State University elaborated an ultrasound test for Dawn Syndrome.

Characteristics of the test:

If Not Disease, the test result is negative with probability 1.

If Disease, the test result is positive with probability 0.5. Assume for a fetus the test result is negative. What is the probability that it has the syndrome?

\[ B_1 \text{ – Disease, } P(B_1) = \frac{1}{80} \]
\[ B_2 \text{ – No Disease, } P(B_2) = \frac{79}{80} \]
\[ A \text{ – Test Negative} \]
\[ P(A) = P(B_1) \cdot P(A \mid B_1) + P(B_2) \cdot P(A \mid B_2) = \]
\[ \frac{1}{80} \cdot 0.5 + \frac{79}{80} = \frac{159}{160}. \]

\[ P(B_1 \mid A) = \frac{1/160}{159/160} = 0.0063. \]


1. System A consists of three components connected in series. The probability of failure for the first is 0.12, for the second is 0.09 and for third is 0.11.

a. Find the probability \( P(W) \) of working of the system.
Answer: \( P(W) = 0.88 \times 0.91 \times 0.11 = 0.7127 \)

b. \( P(W^c) = 1 - 0.7127 = 0.2873. \)
2. System B consists of two pairs of components connected in parallel, the components in each of the pairs are connected in series. All the four components have probabilities of failure 0.1.

c. Find the probability of W: the system B works.

\[ P(W) = .81 + .81 - 0.81^2 = 0.9639. \]

**Elementary Problems on Independence.**

Assume the probability of working of a component is 0.9. What is the probability of its failure? (0.1).

Assume two such components are working. What is the probability that both components work? (0.81) None of them works? (0.01)

Assume 6 components are working (each having the probability of working 0.9). What is the probability that all of them fail? (0.1^6=0.000001).
What is the event complementary to \( D = \{ \text{all the six components fail} \} \)? Answer: \( D^c \{ \text{at least one of the components works} \} \); \( P(D^c) = 1 - 0.1^6 = 0.999999 \) (see the previous problem).

Random variables.

Exercise 4.16, p.194. 3 fair coins are tossed. Consider the set of all possible outcomes:
TTT, TTH, THT, THH, HTT, HTH, HHT, HHH.
Assuming that the tosses are independent, then each of the simple events has the probability 1/8.
Let us assign to each of the simple events the number:

\( X = \text{The number of H-s.} \quad (X(TTT)=0, X(TTH)=1, \text{etc.}) \)

\( X \) is a typical example of a random variable.

Possible values of \( X \) and corresponding probabilities are (the table is called the distribution of the random variable \( X \)):

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8</td>
<td>3/8</td>
<td>3/8</td>
<td>1/8</td>
</tr>
</tbody>
</table>
a. Coming back to problem 4.16, what is the probability $P(X=2 \text{ or } X=3)$? (0.5)

b. What is $E(X)$, the expectation of $X$?

$E(X)=x_1*p_1 + \ldots + x_n*p_n = 1.5$

c. What is the variance of $X$?

$V(X) = (x_1-\mu)^2*p_1 + \ldots + (x_n-\mu)^2*p_n = \frac{3}{4}$, Where $\mu$ denotes $E(X)$, the expectation of $X$.

At last let us calculate the same value of variance by use of the shortcut formula:

$V(X) = x_1^2*p_1 + \ldots + x_n^2*p_n - \mu^2 = \frac{3}{4}$.
Exercise 4.12. (p.194) The random variable $X$ has the following probability distribution:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>.1</td>
<td>.2</td>
<td>.4</td>
<td>.2</td>
<td>.1</td>
</tr>
</tbody>
</table>

a. List the possible values of $X$.
b. What value of $X$ is most probable?
c. Find $P(X=7)$.
d. Find $P(X >2)$.
e. Find $E(X)$ and $V(X)$. (Answer: $E(X)=5$, $V(X)=4.8$.)

In e.: The book says $E(X) = 3.2$, but it is 5. Please check!

Other exercises on independence for upcoming Quiz 2.

**Exercise.** According to USA Today, 65% of Americans are overweight or obese. If 5 Americans are chosen at random what is the probability that at least one of them is overweight or obese? \( 1 - (0.35)^5 = 1 - 0.005 = 0.995. \)

**Exercise.** 3 components working independently are connected in parallel. What is the probability that the system works, if the probabilities of working of these components are respectively 0.96, 0.91, 0.80. \((0.99928).\)

**Exercise.** According to *New York Times*, Verizon is the nation’s larger cellular phone service provider, with 36 million subscribers out of total 148.4 million users in US. If 6 users are randomly selected in the street, what is the probability that at least one of them is Verizon subscriber? \((0.81).\)