Reminders: Percentile, z-score
Finish Worksheet questions for Section 6.

Section 7: Methods for Detecting Outliers: Box Plots and z-Scores

An observation (or measurement) that is unusually large or small relative to the other values in a data set is called an outlier. Outliers typically are attributable to one of the following causes:
1. The measurement is observed, recorded, or entered into the computer incorrectly.
2. The measurement comes from a different population.
3. The measurement is correct but represents a rare (chance) event.

The 5-number Summary: For quantitative data, the 5-number summary consists of the smallest value, the quartiles \( Q_1, Q_2 \) (the median), \( Q_3 \), and the largest value.

The interquartile range (IQR) is the distance between the lower and upper quartiles. \( IQR = Q_3 - Q_1 \)

Box Plot (a graphical representation of quantitative data)
1. Draw a rectangle (box) with the ends (hinges) drawn at the lower and upper quartiles \( Q_1 \) and \( Q_3 \). The median data is shown by a line.
2. The points at distances 1.5(IQR) from each hinge define the inner fences of the data set. Line (whiskers) are drawn from each hinge to the most extreme measurements inside the inner fences.
3. We will not work with outer fences and will refer to inner fences simply as fences.

Rules of Thumb for Detecting Outliers
Box Plots: We will call observations falling beyond the fences ”outliers”.
z-scores: Observations with z-scores greater than 3 in absolute value are considered outliers.

Chapter 3: Probability

Antoine Gombaud, Chevalier de Mere (1607 – 29 December 1684)

Problems of Points
The problem concerns a game of chance with two players who have equal chances of winning each round. The players contribute equally to a prize pot, and agree in advance that the first player to have won a certain number of rounds will collect the entire prize. Now suppose that the game is interrupted by external circumstances before either player has achieved victory. How does one then divide the pot fairly? It is tacitly understood that the division should depend somehow on the number of rounds won by each player, such that a player who is close to winning will get a larger part of the pot.

Which of these two is more probable?
Getting at least one six with four throws of a die;
Getting at least one double six with 24 throws of a pair of dice?
The self-styled Chevalier de Mere believed the two to be equiprobable, based on the following reasoning:
1. A pair of sixes on a single roll of two dice is the same probability as that of rolling two sixes on two rolls of one die.
2. The probability of rolling two sixes on two rolls is 1/6 as likely as one six in one roll.
3. To make up for this, a pair of dice should therefore be rolled six times for every one roll of a single die in order to get the same chance of a pair of sixes.
4. Therefore, rolling a pair of dice six times as often as rolling one die should equal the probabilities.
5. So rolling 2 dice 24 times should result in as many double sixes as getting 1 six throwing 4 dice.
However, betting on getting 2 sixes when rolling 24 times, he lost consistently.
Section 1: Events, Sample Space, and Probability

An **experiment** is an act or process of observation that leads to a single outcome that cannot be predicted with certainty.

A **sample point** is the most basic outcome of an experiment.

The **sample space** of an experiment is the collection of all its sample points.

An **event** is a specific collection of sample points. Further, a **simple event** contains only a single sample point, while a **compound event** contains two or more sample points.

The **probability** of an event A is the numerical measure of the likelihood that the event will occur. It is calculated by summing the probabilities of the sample points in the sample space for A.

**Probability Rules for Sample Points**

Let \( p_i \) represent the probability of sample point \( i \).

1. All sample point probabilities must lie between 0 and 1 (i.e., \( 0 \leq p \leq 1 \)).
2. The probabilities of all the sample points within a sample space must sum to 1 (i.e., \( \Sigma p_i = 1 \)).

**Equally Likely Probability**

\[ P(\text{Event}) = \frac{X}{T} \]

\( X \) = Number of outcomes in the event

\( T \) = Total number of sample points in Sample Space

Each of \( T \) sample points is equally likely, that is, \( P(\text{sample point}) = \frac{1}{T} \).

**Steps for Calculating Probability**

1. Define the experiment; describe the process used to make an observation and the type of observation that will be recorded.
2. List the sample points.
3. Assign probabilities to the sample points.
4. Determine the collection of sample points contained in the event of interest.
5. Sum the probabilities of the sample points in the event to get the ”event probability”. This is the most basic way to determine the probability of an event. We will learn rules that help us determine the probability of an event without doing all this work.

**Combinations Rule**

A sample of \( n \) elements is to be drawn from a set of \( N \) elements. Then, the number of different samples possible is denoted by \( \binom{N}{n} \) and is equal to

\[ \binom{N}{n} = \frac{N!}{n!(N-n)!} \]

where the factorial symbol (!) means that \( n! = n(n-1)(n-2)\ldots(3)(2)(1) \).

For example, \( 5! = 5 \times 4 \times 3 \times 2 \times 1 \). [Note: The quantity 0! is defined to be equal to 1].

Section 2: Unions and Intersections

The **union** of two events A and B is the event that occurs if either A or B or both occur on a single performance of the experiment. We denote the union of events A and B by the symbol \( A \cup B \).

\( A \cup B \) consists of all the sample points that belong to A or B or both.

The **intersection** of two events A and B is the event that occurs if both A and B occur on a single performance of the experiment. We write \( A \cap B \) for the intersection of A and B.

\( A \cap B \) consists of all the sample points belonging to both A and B.

Section 3: Complementary Events

The **complement** of an event A is the event that A does not occur – that is, the event consisting of all sample points that are not in event A. We denote the complement of A by \( A^c \).

**Rule of Complements**

The sum of the probabilities of complementary events equals 1: \( P(A) + P(A^c) = 1 \).