Problem 1:
Toss a coin three times. Let \( H_i \) denote the event of getting a head and \( T_i \) the event of getting a tail at the \( i^{th} \) toss for \( i = 1, 2, 3 \).

(i) Which of the following events is the event of getting at least one head in the three tosses?
(A) \( H_1 \cap H_2 \cap H_3 \)  
(B) \( T_1 \cap T_2 \cap T_3 \)  
(C) \( H_1 \cup H_2 \cup H_3 \)  
(D) \( T_1 \cup T_2 \cup T_3 \)

(ii) List the outcomes (sample points) that are in that event and find the probability of the event.

(iii) The complement of getting at least one head is:
(A) Getting all tails  
(B) Getting all heads  
(C) \( T_1 \cup T_2 \cup T_3 \)  
(D) Getting at least one tail

(iv) List the outcomes (sample points) that are in that event and find the probability of the event.

Problem 2:
One of the words in the sentence "THIS IS EASY" is picked at random and then a letter is picked at random from the word. Let \( F \) stand for "a four letter word is picked", \( S \) stand for "the letter S is picked" and \( Y \) stand for the event "the letter Y is picked". Which of the following is correct?
(A) \( F, S \) are mutually exclusive  
(B) \( \neg F, Y \) are mutually exclusive  
(C) \( F, Y \) are mutually exclusive

Problem 3:
Here is the Venn Diagram of a sample space consisting of 9 outcomes (sample points). Events A, B, and C are depicted.

(i) Suppose that all the outcomes are equally likely. Find the following probabilities:
\[
P(A) \quad P(B) \quad P(A \cap B) \quad P(A \cup B) \quad P(A^c \cap B) \quad P(A \cup B \cup C) \quad P(A^c \cap B^c \cap C^c)
\]

(ii) Now drop the assumption that the outcomes are equally likely. Here are the probabilities of each outcome:
\[
P(o_1) = .10, \quad P(o_2) = .05, \quad P(o_3) = .04, \quad P(o_4) = .06, \quad P(o_5) = .30, \quad P(o_6) = .03, \quad P(o_7) = .07, \quad P(o_8) = .25, \quad P(o_9) = .10
\]
Compute the same probabilities as in part (i).

What do you notice?
Problem 4:
For events A, B, and C, \( P(A) = .5, P(B) = .2, P(C) = .2, P(A \cup B) = .65 \) and \( P(A \cup B \cup C) = .7 \).
We also know that B and C are mutually exclusive. Find the following probabilities:
(i) \( P(A \cap B) \)  
(ii) \( P(B \cup C) \)  
(iii) \( P(A^c \cap B^c) \)  
(iv) \( P(A^c \cap B^c \cap C^c) \)

Problem 5:
Suppose that 0.25 of the population in an area is exposed to a television commercial for Ford automobiles and 0.5 is exposed to Ford’s radio advertisements. Also 0.15 is exposed to both the means of advertising. For a person picked at random from the population, let A stand for the event ”exposed to television commercial” and B stand for ”exposed to radio advertisement”.

(i) What is \( P(A \cap B) \)?
(A) .850  
(B) .75  
(C) .150

(ii) Find the probability of the complement of A.
(A) .150  
(B) .750  
(C) .850  
(D) .600

(iii) What is \( P(A \cup B) \)?
(A) .850  
(B) .600  
(C) .150  
(D) .750

Problem 6:
As part of the language requirement at MSU, 36 \% of the students take French, 27% Spanish, and 50\% take either French or Spanish or both.

(i) What is the percentage of students who take French and Spanish ?

(ii) Are ”the student takes French” and ”the student takes Spanish” mutually exclusive events ?

Problem 7:
The overnight shipping business has skyrocketed in the last ten years. The single greatest predictor of a company’s success is customer service. A study was conducted to determine the customer satisfaction levels for one overnight shipping business. In addition to the customer’s satisfaction level, the customers were asked how often they used overnight shipping. The results are shown below in the following table:

<table>
<thead>
<tr>
<th>Frequency of Use</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2 per month</td>
<td>250</td>
<td>140</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2 – 5 per month</td>
<td>140</td>
<td>55</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>&gt; 5 per month</td>
<td>70</td>
<td>25</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A customer is chosen at random.

(i) What is the probability that the customer has a high satisfaction level ?

(ii) What is the probability that the customer uses overnight shipping at least 2 times per month ?

(iii) What is the probability that the customer uses overnight shipping less than twice a month and has a medium satisfaction level ?

(iv) What is the probability that the customer uses overnight shipping 2 to 5 times per month or has a high satisfaction level ?