In most situations in real life, the true mean and standard deviation of a population are unknown quantities that have to be estimated.

**Definition:** A parameter is a numerical descriptive measure of a population. Because it is based on the observations in the population, its value is almost always unknown.

*Example:* \( p \), the probability of a success in a binomial experiment, and \( \mu \) and \( \sigma \), the mean and standard deviation respectively, of a normal distribution, are examples of parameters.

If you draw a sample from the population of interest, you can always find the sample mean and sample standard deviation.

**Definition:** A sample statistic is a numerical descriptive measure of a sample. It is calculated from the observations in the sample.

*Example:* The sample mean \( \bar{x} \), sample variance \( s^2 \), and sample standard deviation \( s \) are sample statistics.

We will often use the information contained in these sample statistics to make inferences about the parameters of a population.

Note that the term statistic refers to a sample quantity and the term parameter refers to a population quantity.

**Table: Notation for Population Parameters and Corresponding Sample Statistics**

<table>
<thead>
<tr>
<th>Population Parameter</th>
<th>Sample Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ( \mu )</td>
<td>( \bar{x} )</td>
</tr>
<tr>
<td>Variance ( \sigma^2 )</td>
<td>( s^2 )</td>
</tr>
<tr>
<td>Standard Deviation ( \sigma )</td>
<td>( s )</td>
</tr>
<tr>
<td>Binomial proportion ( p )</td>
<td>( \hat{p} )</td>
</tr>
</tbody>
</table>

**Section 1: The Concept of a Sampling Distribution**

*Definition:* The sampling distribution of a sample statistic calculated from a sample of \( n \) measurements is the probability distribution of the statistic.
Section 3: The Sampling Distribution of the Sample Mean and the Central Limit Theorem

Properties of the Sampling Distribution of $\bar{x}$

1. Mean of the sampling distribution equals mean of sampled population, that is, $\mu_{\bar{x}} = E[\bar{x}] = \mu$
2. Standard deviation of the sampling distribution equals \( \frac{\text{Standard deviation of sampled population}}{\sqrt{n}} \).
   That is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
   The standard deviation $\sigma_{\bar{x}}$ is often referred to as the standard error of the mean.

Theorem: If a random sample of $n$ observations is selected from a population with a normal distribution, the sampling distribution of $\sigma_{\bar{X}}$ will be a normal distribution.
If the population is $N(\mu, \sigma)$, then $\bar{X}$ will be $N(\mu, \sigma \sqrt{n})$.

Central Limit Theorem
Consider a random sample of $n$ observations selected from a population (any probability distribution) with mean $\mu$ and standard deviation $\sigma$. Then, when $n$ is sufficiently large, the sampling distribution of $\bar{X}$ will be approximately a normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. The larger the sample size, the better will be the normal approximation to the sampling distribution of $\bar{X}$.

Moreover, because of the Central Limit Theorem, the sum of a random sample of $n$ observations, $\Sigma x$, will possess a sampling distribution that is approximately normal for large samples. This distribution will have a mean equal to $n\mu$ and a variance equal to $n\sigma^2$.

Section 4: The Sampling Distribution of the Sample Proportion

Sampling Distribution of $\hat{p}$

1. Mean of the sampling distribution is equal to the true binomial proportion, $p$; that is, $E(\hat{p}) = p$.
   Consequently, $\hat{p}$ is an unbiased estimator of $p$.
2. Standard deviation of the sampling distribution is equal to $\sqrt{\frac{p(1-p)}{n}}$; that is, $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
3. For large samples, the sampling distribution is approximately normal. (A sample is considered large if $n\hat{p} \geq 15$ and $n(1 - \hat{p}) \geq 15$.)