Section 9.1

5.

a. $H_a$ says that the average calorie output for sufferers is more than 1 cal/cm$^2$/min below that for non-sufferers. \[
\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}} = \sqrt{\frac{(2)^2}{10} + \frac{(4)^2}{10}} = .1414, \text{ so}
\]
\[
z = \frac{(6.4 - 2.05) - (-1)}{.1414} = -2.90. \text{ At level .01, } H_0 \text{ is rejected if } z \leq -2.33; \text{ since } -2.90 < -2.33, \text{ reject } H_0.
\]

b. \[P = \Phi(-2.90) = .0019\]

c. \[
\beta = 1 - \Phi\left(-2.33 - \frac{-1.2 + 1}{.1414}\right) = 1 - \Phi(-.92) = .8212
\]

d. \[m = n = \frac{2(2.33 + 1.28)^2}{(-.2)^2} = 65.15, \text{ so use 66.}\]

7.

1. Parameter of interest: $\mu_1 - \mu_2 = \text{the true difference of means for males and females on the Boredom Proneness Rating. Let } \mu_1 = \text{men’s average and} \mu_2 = \text{women’s average.}
2. $H_0: \mu_1 - \mu_2 = 0$
3. $H_a: \mu_1 - \mu_2 > 0$
4. \[
z = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}} = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}
\]
5. RR: $z \geq 1.645$
6. \[
z = \frac{(10.40 - 9.26) - 0}{\sqrt{\frac{4.83^2}{97} + \frac{4.68^2}{148}}} = 1.83
\]
7. Reject $H_0$. The data indicates the average Boredom Proneness Rating is higher for males than for females.
11. \[ (\bar{X} - \bar{Y}) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}. \] Standard error = \[ \frac{s}{\sqrt{n}}. \] Substitution yields \[ (\bar{X} - \bar{Y}) \pm z_{\alpha/2} \sqrt{(SE_1)^2 + (SE_2)^2}. \] Using \( \alpha = 0.05 \), \( z_{\alpha/2} = 1.96 \), so \( (5.5 - 3.8) \pm 1.96\sqrt{(0.3)^2 + (0.2)^2} = (0.99, 2.41) \). We are 95% confident that the true average blood lead level for male workers is between 0.99 and 2.41 higher than the corresponding average for female workers.

Section 9.2

19. For the given hypotheses, the test statistic \[ t = \frac{115.7 - 129.3 + 10}{\sqrt{\frac{5.0\cdot^2}{6} + \frac{5.3\cdot^2}{6}}} = \frac{-3.6}{3.007} = -1.20, \] and the d.f. is \[ v = \frac{(4.2168 + 4.8241)^2}{(4.2168)^2 + (4.8241)^2} = 9.96, \] so use d.f. = 9. We will reject \( H_0 \) if \[ t \leq -t_{0.19} = -2.764; \] since \(-1.20 > -2.764\), we don’t reject \( H_0 \).

22. Let \( \mu_1 = \) the true average strength for wire-brushing preparation and let \( \mu_2 = \) the average strength for hand-chisel preparation. Since we are concerned about any possible difference between the two means, a two-sided test is appropriate. We test \( H_0 : \mu_1 - \mu_2 = 0 \) vs. \( H_a : \mu_1 - \mu_2 \neq 0 \). We need the degrees of freedom to find the rejection region: \[ v = \frac{1.58^2 + 4.0^2}{11} = \frac{2.3964}{0.0039 + 1.632} = 14.33, \] which we round down to 14, so we reject \( H_0 \) if \( |t| \geq t_{0.025,14} = 2.145 \). The test statistic is \[ t = \frac{19.20 - 23.13}{\sqrt{\frac{1.58^2 + 4.0^2}{12}}} = -3.93 \] and \[ 1.2442 = -3.159, \] which is \( \leq -2.145 \), so we reject \( H_0 \) and conclude that there does appear to be a difference between the two population average strengths.
30.

a. We desire a 99% confidence interval. First we calculate the degrees of freedom:

\[
\nu = \frac{\left(\frac{2.3^2}{26} + \frac{4.3^2}{26}\right)^2}{\frac{2.3^2}{26} + \frac{4.3^2}{26}} = 37.24 ,
\]

which we would round down to 37, except that there is no \( \nu = 37 \) row in Table A.5. Using 36 degrees of freedom (a more conservative choice), \( t_{0.05} = 2.719 \), and the 99% C.I. is

\[
(33.4 - 42.8) \pm 2.719 \sqrt{\frac{2.3^2}{26} + \frac{4.3^2}{26}} = -9.4 \pm 2.576 = (-11.98, -6.83) .
\]

We are 99% confident that the true average load for carbon beams exceeds that for fiberglass beams by between 6.83 and 11.98 kN.

b. The upper limit of the interval in part a does not give a 99% upper confidence bound. The 99% upper bound would be

\[
-9.4 + 2.434(9.473) = -7.09 ,
\]

meaning that the true average load for carbon beams exceeds that for fiberglass beams by at least 7.09 kN.