(1) Let \( N = 1, 2, \ldots \) and \( \Theta = \{ (a, b) : a \leq b, a, b \in N \} \).
For each \( (a, b) \in \Theta \), let \( P(a) = P(b) = .5 \)
Let \( X_1, X_2, X_3 \) be i.i.d \( P_{a,b} \).
Let
\[ T(X_1, X_2, X_3) = (X_1 + X_2 + X_3)/3 \]
\[ S(X_1, X_2, X_3) = [\text{Max}(X_1, X_2, X_3) + \text{Min}(X_1, X_2, X_3)]/2 \]
Is either of the above a MVUE of \( g(a, b) = (a + b)/2 \). If not find the
MVUE.
Clearly state the results you use and give the details of your argument
clearly. [15]

(2) Suppose that \( \{ f_{\theta} : \theta \in \Theta \} \) is a family of densities such that for all \( n \)
there exists a MVUE estimate based for \( \theta \) based on i.i.d observations
\( X_1, X_2, \ldots, X_n \). If \( V_{n,\theta} \) is finite for some \( n \), show that \( V_{n,\theta} \) goes to 0 as
\( n \to \infty \) [8]

(3) Let \( \phi \) be a MP size \( \alpha \) test function for testing \( H_0 : P \) against \( H_1 : Q \).
Prove or give a counter example to the following statement:
\"\( \phi \equiv \alpha \) a.e. \( P + Q \) iff \( P = Q \).\" [10]

(4) Consider the testing problem \( H_0 : \theta \in \{ -1, 1 \} \) against the alternative
\( H_1 : \theta = 0 \) where \( \theta \) is the mean of a normal population with variance 1.
(a) show that there does not exist a UMP test for the above problem
   based on one observation. [Begin by showing that such a test has to be
   symmetric] [10]
(b) Does there exist a UMP test based on \( n \) observations for some \( n \)?
   Justify your answer. [7]

(5) Let \( X \) have the distribution function
\[ F_\alpha(x) = 1 - x^{-\alpha}, \quad \alpha > 0 \]
(a) Show that for \( \alpha > 2, \)
\[ E(X) = \frac{\alpha}{(\alpha - 1)} \quad \text{and} \quad V(X) = \frac{\alpha}{(\alpha - 2)(\alpha - 1)^2} \]
[5]
(b) Let \( Y = (\alpha - 1)X - \alpha \). Show that as \( \alpha \to \infty \), \( Y \) converges in
distribution and identify the limit. [15]

(6) Suppose \( X_1, X_2, \ldots, X_n \) is a sample from \( U(0, \theta) \). The MLE of \( \theta \) is \( M_n \)-
the maximum of \( X_1, X_2, \ldots, X_n \).
(a) Show that \( n(\theta - M_n) \) converges in distribution to an exponential dis-
   tribution [10]
(b) In view of the above, as an estimate of \( \theta \), \( M_n \) might not be so good
   since \( M_n < \theta \) with probability 1. Consider the modified estimate
   \[ T_n = \frac{(n+c)}{n} M_n. \]
   (i) what is the asymptotic distribution of \( T_n \) [10]
   (ii) What value of \( c \) should be used if we measure accuracy by
       squared error loss? absolute error loss? [10]
(7) Let \(X_1, X_2, \ldots\), be i.i.d random variables with mean \(\mu\) and variance \(\sigma^2\). Find the asymptotic distribution of
\[
R_n = \frac{\sum_{i=1}^{n} X_{2i-1}}{\sum_{i=1}^{n} X_{2i}}
\]
[Deal with the case \(\mu = 0\) and \(\mu \neq 0\) separately][15]

(8) For \(i = 1, 2, \ldots\), let
\[
Y_i = \theta x_i + \epsilon_i
\]
where \(\epsilon_i, i = 1, 2, \ldots\), are i.i.d symmetric variables and \(x_1, x_2, \ldots\) are non random design values.

Give some sufficient conditions on the model which will ensure the consistency of the least square estimates of \(\theta\) as \(n\) goes to \(\infty\).[15]

(9) Let \(X_1, X_2, \ldots, X_n\) be i.i.d observations from \(U(0, \theta)\) where \(\theta \in \Theta = \{1, 2, 3, \ldots\}\).
(a) find the MLE of \(\theta\) [10]
(b) Investigate the admissibility of the MLE for 0-1 loss [10]
(c) what can you say about \(\sup_\theta R(\theta, T)\) where \(T\) is the MLE. [5]