1. Let \( X_1, \ldots, X_n, n > 2 \), be iid observations from the exponential distribution \( E(0, e^\theta), \theta \in \mathbb{R} \). Show that the MLE of \( \theta \) is \( \hat{\theta} = \log \bar{X} \) and explicitly find the \( \sigma^2_\theta > 0 \) such that \( \sqrt{n}(\hat{\theta} - \theta) \to N(0, \sigma^2_\theta) \), as \( n \to \infty \).  

2. Let \( \Theta \) denote the set of integers \( \{2, 3, \ldots\} \). Let \( X_1, \ldots, X_n \) be iid observations from 
\[
 f_\theta(x) = \theta (1 - x)^{\theta-1} I(0 \leq x \leq 1), \quad \text{for some } \theta \in \Theta. 
\]

(a) Find the MLE \( \hat{\theta} \) of the true parameter \( \theta \).  
(b) Show that \( \hat{\theta} \) is consistent for \( \theta \).

3. Let \( \Theta := \{ (\theta_1, \theta_2, \mu); \theta_1 > 0, \theta_2 > 0, \mu \in \mathbb{R} \} \). Let 
\[
 f_{\theta_1, \theta_2, \mu}(x) = \begin{cases} 
 (\theta_1 + \theta_2)^{-1} e^{-(x-\mu)/\theta_1} & x \geq \mu \\
 (\theta_1 + \theta_2)^{-1} e^{(x-\mu)/\theta_2} & x < \mu 
\end{cases}, \quad (\theta_1, \theta_2, \mu) \in \Theta. 
\]

(a) Show that the family of Lebesgue probability densities \( \{ f_{\theta_1, \theta_2, \mu}(x); (\theta_1, \theta_2, \mu) \in \Theta \} \) is complete.  
(b) Show that UMVUE for \( \mu \) based on a single observation \( X \) from this density does not exist in general. Find a subset of \( \Theta \) by imposing condition on \( \theta_1, \theta_2 \) such that UMVUE for \( \mu \) based on \( X \) exists.

4. Let \( X \) be a single observation from \( N(\mu, 1) \) where \( \mu \in \mathbb{R} \) has the improper Lebesgue prior density \( \pi(\mu) = e^\mu \). Under the squared error loss, show that the generalized Bayes estimator of \( \mu \) is \( X + 1 \), and that it is neither minimax nor admissible.