COLLOQUIUM

Department of Statistics and Probability
Michigan State University

Han Xiao
University of Chicago

ON SAMPLE AUTOCOVARIANCES OF STATIONARY PROCESSES

Tuesday, November 23, 2010
A405 Wells Hall
10:20 a.m. - 11:10 a.m.
Refreshments: 10:00 a.m.

Abstract

Testing for serial correlation has been extensively studied in both statistics and econometrics. Let $(X_i)_{i\in\mathbb{Z}}$ be a real-valued stationary process. Denote by γ_k and r_k the autocovariance and autocorrelation at lag k, and by $\hat{\gamma}_k$ and $\hat{r}_k$ the corresponding estimates. The Box-Pierce portmanteau test uses $Q_K = n\sum_{k=1}^{K}\hat{r}_k^2$ as the test statistic, and rejects if it lies in the upper tail of χ_K^2 distribution. An arguable deficiency of this test is that the number of lags K included in the test is held as a constant in the asymptotic theory. The problem is particularly relevant if practitioners have no prior information about the alternatives. Hong (1996) established the following result:

$$\frac{1}{\sqrt{2s_n}}\left(n\sum_{k=1}^{s_n}(\hat{r}_k - r_k)^2 - s_n\right) \Rightarrow \mathcal{N}(\prime, \infty), \tag{1}$$

under the condition that (X_i) is an iid sequence.

Another natural omnibus choice is to use the maximal autocorrelation as the test statistic. Wu (2009) obtained a stochastic upper bound for

$$\sqrt{n}\max_{1\le k\le s_n}|\hat{\gamma}_k - \gamma_k|, \tag{2}$$

and argued that the test based on (2) has higher power over the Box-Pierce tests with unbounded lags in detecting weak serial correlation.

If one wants to go further from (1) and (2), one may naturally ask: (i) what if the serial correlation is present in (1); and (ii) what is the asymptotic distribution of (2)? We show that (1) remains true even if all or some of r_k are not zero, but the normalizing constants, being different, will depend on the values of r_k. Furthermore, we also find the limiting distribution of $\sum_{k=1}^{s_n}\hat{r}_k^2$ when the serial correlation is present, which enables us to calculate the asymptotic power of the Box-Pierce test with unbounded lags. Question (ii) is more challenging, we show that the quantity in (2) converges to the Gumbel distribution after suitable normalization.

The asymptotic tests often perform poorly when the sample size n is not large enough. The problem is particularly relevant for tests based on (2), as the convergence to the Gumbel distribution is known to be very slow. Our simulation results suggest that block of blocks bootstrapping provide a reasonably good approximation to the distribution of (2) for small samples.

If time permits, I will also talk about the estimation of autocovariance matrices.