Note Title 9/28/2009

STT 200 9-28-09

TODAY: CH 6. NORMAL DISTRIBUTIONS. (BELL).

(1) APPEARANCE OF BELL CURVE; IDENTIFYING MEAN, SD.

(2) STANDARD SCORES (SCORE → z → SCORE w/ MEAN, SD GIVEN)

(3) USE OF NORMAL NOTATION $N(\mu, \sigma^2)$

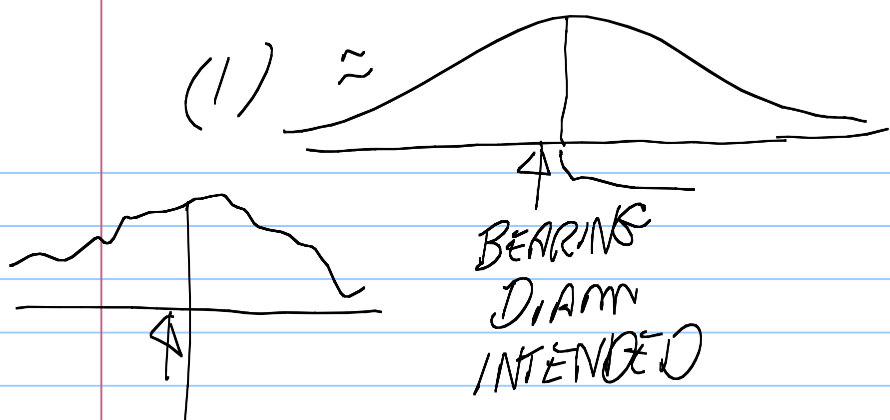
(4) USE OF z TABLE ($N(0,1)$). *

SCORE x, μ_x, σ_x → $z = \dfrac{x - \mu_x}{\sigma_x}$

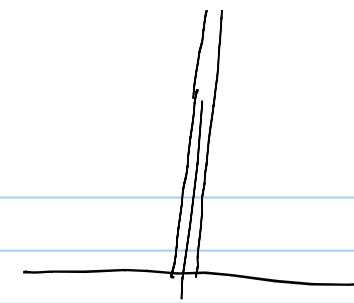
z, μ_x, σ_x → $x = \mu + z\,\sigma_x$.

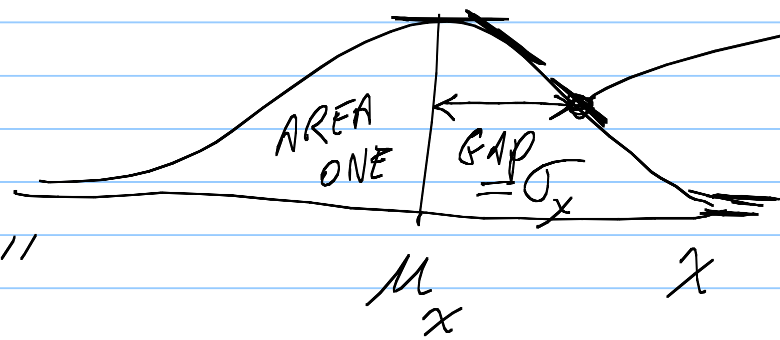
(5) PROPERTIES OF NORMAL.

(1) $\approx$



$\approx$





"mu"

POINT OF INFLECTION

eg IQ $\mu_x = 100$, $\sigma_x = 15$.

$\approx$

$\therefore$ SAY eg $P([85, 115])$ IQ = AREA UNDER CURVE BETWEEN 85, 115

JUST $\mu_x \pm 1\sigma_x$

RULE OF THUMB 68%
(0.68)

REALIZE — NORMAL ARE THEORETICAL IDEALIZATIONS —

<u>DENSITY</u> $\Pr(\text{IQ } 85) = 0$ IDEAL

NO PARTICULAR IQ SCORE HAS POSITIVE PROB^y UNDER THE IDEAL.

TECHNIC:

$$\left(\frac{1}{\sigma_x\sqrt{2\pi}}\right) e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}$$

$e = 2.718281828\ldots$

BUT WE DON'T USE THIS FORMULA IN THE COURSE.

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(2) STD SCORES. eg IQ = 114 $\mu_x = 100$ $\sigma_x = 15$

"i.e. zee" $z = \frac{114-100}{15} = \frac{14}{15}$ STANDARD SCORE OF IQ 114.

REVERSE: IF MY z SCORE IS $-.3$ $z = -.3$

$\Rightarrow$ MY IQ IS $100 - (.3)15 = 100 - 4.5$ IQ.

— STANDARD SCORES ARE UNITLESS —

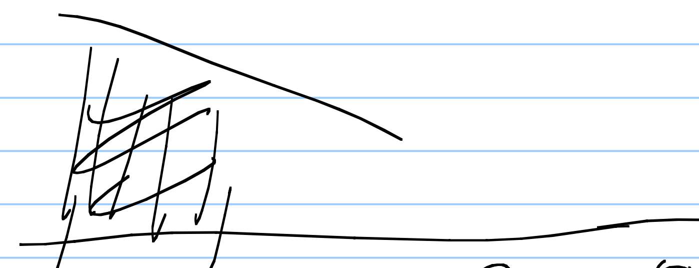
BEAUTIFUL THING: ALL NORMALS ARE ALIKE IN AREA CAPTURED BETWEEN POINTS HAVE SAME STD SCORE.

IQ

100 100+(.6)15

IQ

SAME AREA AS



.25 .25+(.6).001

BEARING DIAM

IQ $\mu_x = 100$ BEARING DIAM $\mu_y = .25$

$\sigma_x = 15$ $\sigma_y = .001$

BELL

ALL NORMALS ALIKE IS STD DEV UNITS FROM MEAN.

IN BOTH EXAMPLES STD SCORES ARE 0 AND 0.6

STD SCORE OF μ_x μ_y — OTHER STD SCORE

(3) NOTATION: σ^2 CALLED VARIANCE. $N(\mu, \sigma^2)$

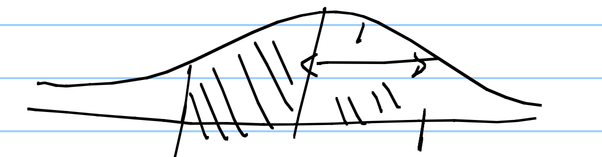
$N(100, \underbrace{225}_{\text{SQ OF } \sigma_{IQ} = 15})$

(4) USE OF z TABLES.

Standard (TABLE)

FIND THE AREA (UNDER z)
BETWEEN $[-1.83, +1.69]$

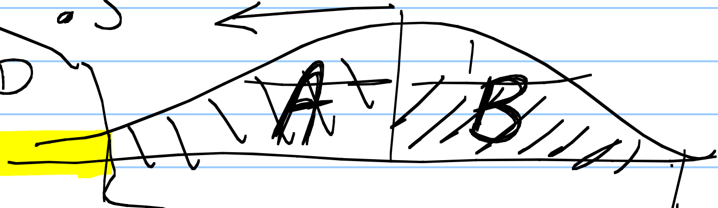
STD NORMAL
$N(0,1)$



-1 0 1 z

$\underbrace{\qquad\qquad}_{68\%}$

.5 ←



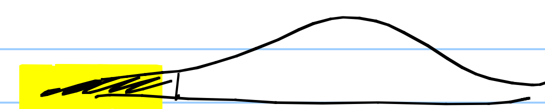
FOUND TO BE .0336

-1.83 0 1.69 z

A + B

$A = .5 -$ AREA LEFT OF -1.83

FIND A



z

$-1.83 \rightarrow .03$

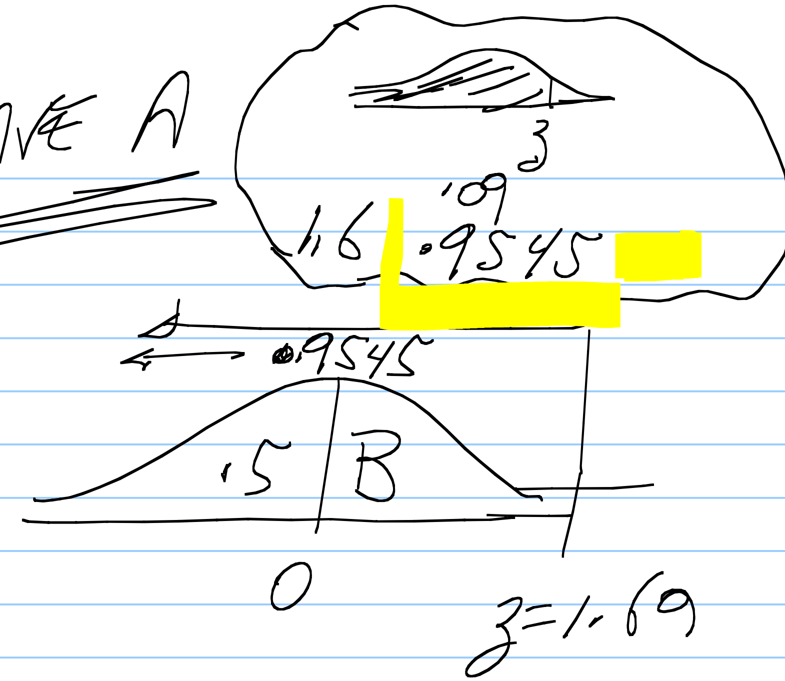
-1.8

$\boxed{0.0336}$

SO $A = .5 - .0336$

AREA BELOW 0 — AREA BELOW -1.83

HAVE A

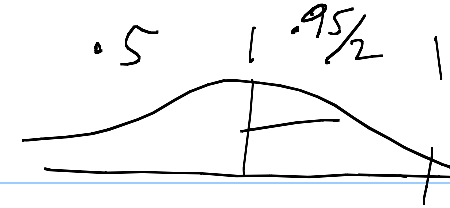


NOW $B =$ AREA LEFT OF 1.69 $- .5$

$A + B =$ ANS. TO QUESTION

$P([-1.83, 1.69])$ IN z DISTRIBUTION

ANOTHER EXAMPLE:



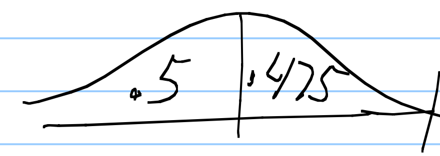
IQ $\mu_x = 100$ $\sigma_x = 15$

WHAT % OF POPULATION HAVE IQ < 130

ANS. .975 BY 2 SD RULE OF THUMB.

OR "

USE TABLE OF z.



WANT PR z BELOW 2.00

.00

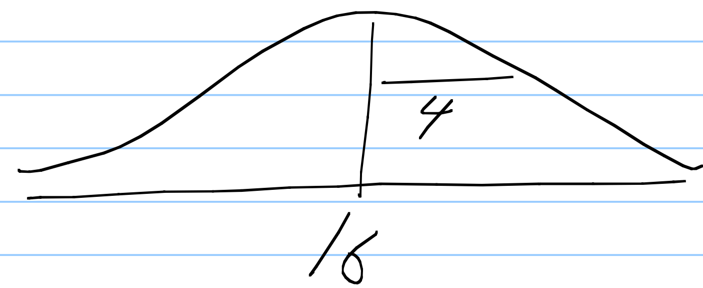
2.0 .9772

SO $P(IQ < 130)^{\text{THEORY}} = .9772$

STILL ANOTHER.

OF RARE EVENTS.

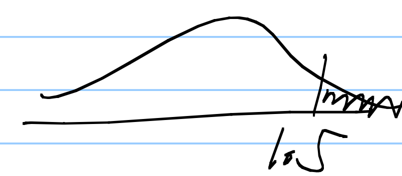
SAY WE AVG $\approx$ 16 ACCIDENTS / YR.

$\sim$ 

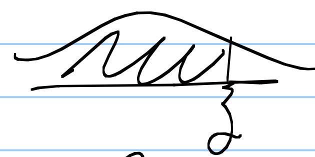
? P(MORE THAN 22 ACCIDENTS IN A GIVEN YR)

STD SCORE OF 22 IS $\frac{22-16}{4} = 6/4 = 3/2 = 1.5$

P(> 22) = P(TO RIGHT OF 1.5 IN z)



ANS: $1 -$ AREA LEFT OF $z = 1.5$



z	$.00$
1.5	$.9332$

SO $P(\text{GET} > 22 \text{ ACCIDENTS})$

$\approx 1 - .9332$

$= .0668$

VERY SPECIAL CASE WHERE $\sigma = \sqrt{\mu}$

CONTEXT OF COUNTS OF RARE EVENTS.

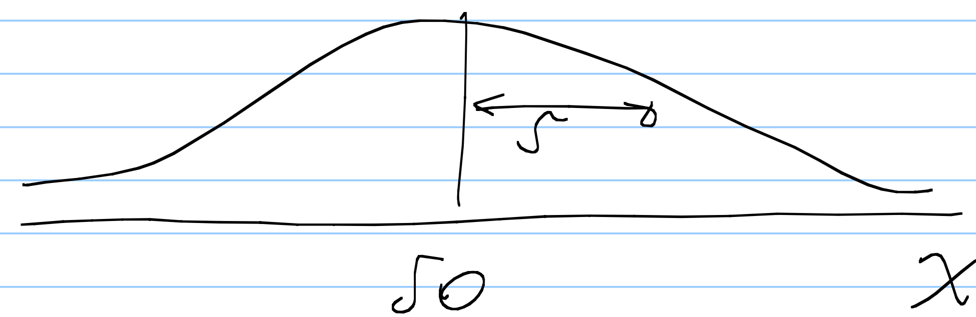
ANOTHER EXAMPLE. TOSS COIN 100 TIMES

$X = \#$ HEADS.

$\mu_X = 50$ FORMULA FOR σ_X GIVES 5 $\sigma_X = 5$

$\approx$ DIST OF x = # HEADS IN 100 TOSSES

$\approx$



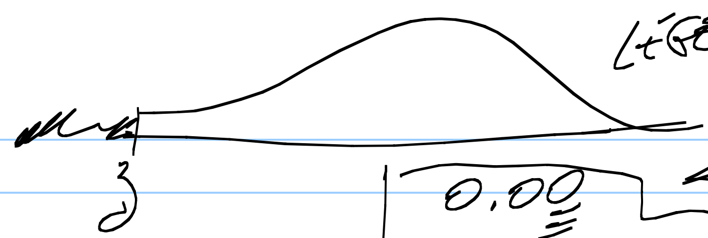
? P(GET FEWER THAN 35 H)

$\approx P(z < \text{STD SCORE OF } 35) = P\left(z < \dfrac{35 - 50}{5}\right)$

μ_x

x

$= P(z < -15/5) = P(z < -3.00)$



LEGEND

CODE FOR -3.00

TABLE

0.00

.0013 -3.0

$P(<35H) \approx .0013$