STT 200  10-28-09

Today

1. Walk Jarvis CI \( p^* \pm 1.96 \frac{\text{UP} - \text{LO}}{\sqrt{n}} \)

Population = Those Present

\[ N = 30 \] (Present)

Random Sample for Purpose of Estimating \( p \) = Fraction of Pop who are Junior or Senior,

\[ p = \frac{10}{30} = \frac{1}{3} \] NOT usually seen.

Draw Random Sample of Population without Replacement Sample of \( n = 11 \)

Sample consists of those having (last digit) of student # = 0 or 1 or 2.
1. We now have 11 people who represent a without replacement random sample of the population of $N = 30$, $n = 11$

2. We find 4 of 11 are junior or senior.

So parameter $\hat{p} = \frac{20}{30}$ (emp)

is in this case estimated from the sample as $\hat{p} = \frac{4}{11}$. (Chance $\hat{p}$ (random) = $p$ slight)

3. A 95% CI prescription $\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

This evaluates to $\frac{4}{11} \pm 1.96 \sqrt{\frac{\frac{4}{11}(1-\frac{4}{11})}{11}}$

4. $P(\hat{p} \in \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}) \approx .95$ as $n \to \infty$ in without-replacement sampling.
5. Since we sampled without replacement the FPC should be used: 
\[ \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{N-m}} \sqrt{\frac{N-n}{N-1}} \]

6. Claim: 
\[ P(\hat{p} \in \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{N-m}} \sqrt{\frac{N-n}{N-1}}) \sim 0.95 \]

Note: May have to pay attention to the assumptions and the nature of sampling.
2. Per Exam 3 (you'll see in Prep 5 I will read) Sure: "Mechanical Acts"

Also: Assumptions. Random sample: $\alpha \rightarrow \sigma$

3. Current material will include stratified sampling (Post-Stratified Sampling).

Idea: Perhaps $P(\alpha = 1.0)$ is post-secondary?

But random sample has $\approx 10^9$

Then we suspect for score

$X = \text{income}$ That $\bar{X}$ is biased down, ??
Sample $n$ from $N = N_1 + N_2$

$K = \#\text{STRATA} = 2$

Grad NS of less Post-secondary

(Subpopulation 1) (Subpopulation 2)

$X = \text{INCOME}$

Usually just sample $n$ (with repl) from entire BP (if non NS stratum) $P(M_x \text{ in } \bar{x} \pm 1.96 \frac{SD}{\sqrt{n}}) \sim 0.95$

$\bar{X} = \frac{m_1}{M} \bar{X}_1 + \frac{m_2}{M} \bar{X}_2$ (because $\frac{X_i + X_{i1} + \ldots + X_{iM_i}}{M_i}$)

$\# \text{ of HS in sample}$ $\bar{X}$ $\text{avg income of } \leq \text{ HS group in sample}$
But compare with \( \mu_x = \frac{N_1}{N} \mu_1 + \frac{N_2}{N} \mu_2 \)

over whole pop \( N \)

If you know \( \frac{N}{N} \) this suggests using

(Instead of \( \bar{X} \)) \( X = \left( \begin{array}{c} \frac{N_1}{N} \bar{x}_1 + \frac{N_2}{N} \bar{x}_2 \\ \end{array} \right) \)

using \( X \) is like using \( \frac{m_1}{n_1} \) and \( \frac{m_2}{n_2} \)

so upshot is another 95% CI for \( \mu_x \)

\[
\frac{X}{\bar{x}} \pm 1.96 \sqrt{\frac{\sum_{i=1}^{2} (N_i)^2}{n_i} \frac{\sigma_x^2}{N}}
\]

\( w_i = \frac{N_i}{N} \)
Note: \( \bar{x} = \frac{\sum_{i=1}^{K} w_i \cdot x_i}{K} \)

\( \bar{x}_i \): Average income for sample individuals in stratum \( i \), \( i = 1, 2 \)

\( s_i \): Sample SD of incomes of sample individuals in stratum \( i \)

Example:

\( x \): Income = Money on hand

\( m = 11 \):

Stratum 1: Women

Stratum 2: Men

1. \# \( x \) 100 0

Income 1

2. \# \( x \) 30 51.2 15 20

Income 2

\[ N_1 = 15 \]

\[ N_2 = 15 - \frac{N_1}{2} \]

\[ N = 30 \]
\[
\text{WOMEN} \quad \mu_1 = \sqrt{\frac{\sum(x_i - \bar{x}_1)^2}{n-1}} = \sqrt{\frac{(100-50)^2 + (0-50)^2}{2-1}} = 10.4 (50)
\]

\[
\bar{x}_1 = \frac{100 + 0}{2} = 50 \quad n_1 = 2
\]

\[
\mu_2 = \sqrt{\frac{(30-20)^2 + \cdots + (20-20)^2}{4-1}}
\]

\[
\bar{x}_2 = \frac{30+1 \cdot 20 + 11 + 20}{4} \sim 29
\]

\[
\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} \quad \frac{n_1}{N} = \frac{15}{30}
\]

\[
\frac{N_1}{N} \bar{x}_1 + \frac{N_2}{N} \bar{x}_2 = \frac{1}{2} \bar{x}_1 + \frac{1}{2} \bar{x}_2 = \frac{1}{2} 50 + \frac{1}{2} 29
\]

\text{REDD STRATIFIED ESTIMATOR}
Est. moe for post-strat is

\[ 1.96 \sqrt{\frac{(1.45)^2}{2} + \frac{??}{4}} \]

\[ \hat{\rho}_1 = 2 \text{ (50)}^2 \]

\[ m_1 = 2 \]

Sample women

What then is stratification?

\[ N = N_1 + N_2 \]

Caution!

\[ m_2 = 4 \]

\[ m_1 = 2 \]

Far too small.!

This was just a walk-through of calculations.
\[ \bar{x} \pm 1.96 \sqrt{\frac{\sum_{i=1}^{N_1} \left( \frac{N_1}{N} \right)^2 (\bar{x}_1 - \bar{x})^2}{N_1}}. \]

Works out - same as \[ \frac{N_1}{N_2} \bar{x}_1 + \frac{N_2}{N} \bar{x}_2 \] since \[ \frac{N_1}{N} = \frac{N_1}{N_1 + N_2} \].