STT 200 11-11-09

Note Title 11/11/2009

TODAY: CH 15

(a) INTERESTING CLASSICAL MODELS

(b) CLASSICAL MODELS $\Rightarrow$ RULES $\Rightarrow$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B \mid \text{IF } A) = P(A \cap B) / P(A)$$

$$P(A \cap B) = P(A)\, P(B \mid \text{IF } A)$$

(c) EXERCISES INVOLVING USE OF RULES OUTSIDE CLASSICAL SETTING.

(d) (POSSIBLY TODAY) BAYES / TREES. (INDUCTION)

(a.) $P(① > ②)$ ← INDICATES $① > ②$

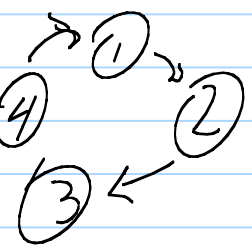
① \ ②	3	4	5	20	21	22
6	≡	≡	≡			
7	≡	≡	≡			
8	≡	≡	≡			
9	≡	≡	≡			
23	≡	≡	≡	≡	≡	≡
24	≡	≡	≡	≡	≡	≡

MAGIC DICE (HOW TO MAKE IT BACK)

① { 6, 7, 8, 9, 23, 24
② { 3, 4, 5, 20, 21, 22
③ { 1, 2, 16, 17, 18, 19
④ { 10, 11, 12, 13, 14, 15

$P(① > ②) = \frac{\#(① > ②)}{36 \text{ POSSIBLE}} = \frac{24}{36} = \frac{2}{3}$

NON TRANSITIVE



→ TOO (CK, IT!) $P(② > ③) = \frac{2}{3}$; $P(③ > ④) = \frac{2}{3}$

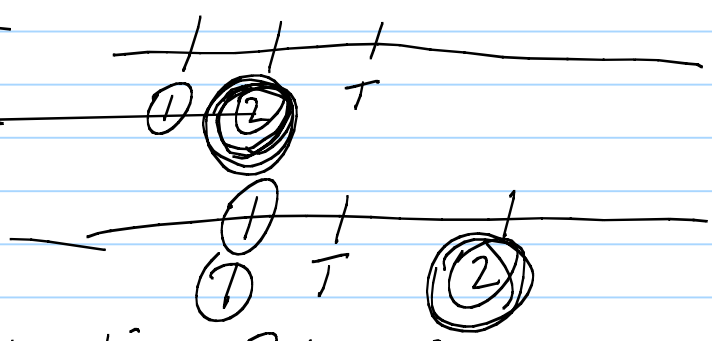
SO / BUT ? $P(① > ④)$? ACTUALLY $P(④ > ①) = \frac{2}{3}$

? BEST TEAM ?

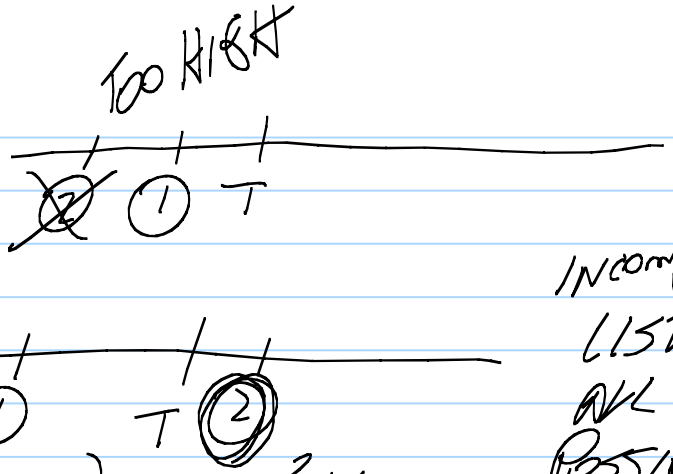
→

EXAMPLE 2. SECOND GUESSING.

MODEL ⇒



TOO HIGH



INCOMPLETE LIST OF ALL POSSIBILITIES ⇒

? Ck. D P(2nd GUESSER WINS) = 3/4

THIS MODEL

EXAMPLE 3. SECRETARY PROBLEM

SKIP FIRST n/e $e \sim 2.718281828$

TAKE FIRST THEREAFTER THAT IS BEST SO FAR

? P(WIND UP WITH THE VERY BEST OF n) $\sim \frac{1}{3}$

RELEVANCE OF CLASSICAL MODELS TO TABLES OF COUNTS.

INCOME \ HEALTH	L	ML	MH	H	MARGINAL TOTALS
L	16	21	43	10	90
ML	18	22	9	40	
H	8	16	31	64	
	99				(n = TOTAL OF ALL 12)

So (CLASSICAL) $P(ML) = 99/n = 99/(16+\cdots+64)$

(n = 12 CELLS)

So TO $P(ML \mid L \text{ INCOME})$

IF (GIVEN) (or)

$$= \frac{21}{16+21+43+10} = \frac{21/n}{(16+\cdots+10)/n}$$

$$= \frac{P(L \text{ INCOME and } (\cap) \text{ ML HEALTH})}{P(L \text{ INCOME})}$$

$$P(\text{ML}_H \mid_{IF} L_{INC}) = P(\text{ML}_H \text{ and } L_{INC}) / P(L_{INC})$$

eg (ANOTHER CONTEXT)

$$P(\text{LEFT HANDED} \mid_{IF} \text{WOMAN}) = \frac{P(L \text{ + WOMAN})}{P(\text{WOMAN})}$$

SIMPLE ARITHMETICAL FEATURE

IF PROBABILITIES APPLY ($\simeq$) TO REAL WORLD COUNTS

THEN PROBABILITIES MUST OBEY

$$P(B \mid_{IF} A) = P(A \text{ and } B) / P(A)$$

"CONDITIONING EVENT"

CONDITIONING EVENT.

$$\Longleftrightarrow P(A \text{ and } B) = P(A)\, P(B \mid_{IF} A)$$

eg | 4R, 6G | DRAW TWO WITHOUT REPL.

$$P(R1 \cap_{\text{and}} G2) = \underbrace{P(R1)}_{4/10}\ P(G2 \mid R1)_{\text{IF}}^{6/9} \quad \text{WITHOUT REPL.}$$

INSTEAD WITH-REPL

$$P(R1 \cap_{\text{and}} G2) = 4/10\ \ 6/10$$

AIRCRAFT: A B

~~A B~~ TWO ENGINES.

A DENOTES "ENGINE A FAILS"

B " "B-FAILS"

$$P(A \cap_{\text{and}} B) = P(A)\,P(B \mid A)_{\text{IF}}$$

IF SPEC SAYS $P(A) = .0001$ SO TOO $P(B) = .0001$

$$? P(A \text{ and } B) = P(A)\, P(B \mid A)$$

$.0001$ — IF — PROBABLY CLOSE TO 1

IF FAILURE OF A IS IN CONJUNCTION WITH FLOCKS OF BIRDS.

INDEPENDENCE

WE SAY A, B ARE STATISTICALLY INDEPENDENT IF $P(B \mid A) = P(B)$

SO KNOWING A DOES NOT CHANGE $P(B)$.

CAN PROVE (EASILY) THAT $P(B \mid \text{NOT } A) = P(B)$

KNOWING OUTCOME OF A DOES NOT ALTER $P(B)$

IN A TABLE

	B	NOT B
A	6	12
$A^c = \bar{A}$ = NOT A	24	48

PROPORTIONAL

APPEARANCE OF INDEPENDENCE IN TABLE OF COUNTS

Eg IF YOU SEE SOMETHING CLOSE TO

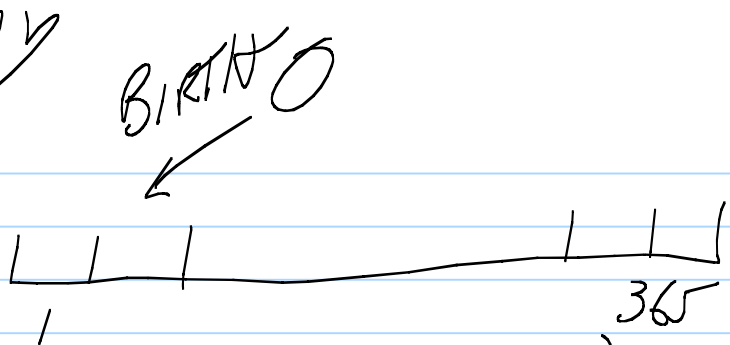
USE CELL LEFT	TUMOR LEFT	TUMOR RIGHT
	642	710
RIGHT	1284	1500

PERFECT INDEPENDENCES

NAME MULT RULE : $P(A \cap \text{and } B) = P(A)\,P(B \mid A)$

$\Rightarrow P(A \text{ and } B \text{ and } C) = P(A)\,P(B \mid \text{IF } A)\,P(C \mid \text{IF } A \text{ and } B)$

BIRTHDAY



$P(n \text{ BIRTHS AVOID EACH OTHER}) = \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \cdots \frac{366-n}{365}$

FOR $n = 23 \approx \frac{1}{2}$

2nd misses 1st

ACTUALLY, $P(\text{NO BIRTH SHARING AMONG } n \text{ PERSONS})$

$\geq < \frac{1}{2}$ BECAUSE SOME DAYS OF THE YEAR ARE MORE LIKELY (BIRTHDAYS)

ACE OF SPADES: $P(n \text{ DRAWS FROM SHUFFLED FULL DECK PRODUCE NO ACE OF SPADES})$

$= \frac{51}{52}\frac{51}{52}\frac{51}{52}$ n-FOLD $= \left(\frac{51}{52}\right)^n = \left(1 - \frac{1}{52}\right)^n \approx e^{-n/52}$ ← LARGE

WITH REPL
INDEP DRAWS.

↑ SMALL

SO IN 52 DRAWS

$P(\text{NO ACE SPADES}) \sim e^{-1} \simeq \frac{1}{3}$.

ON THE OTHER HAND,

IN WITHOUT REPL DRAWS $P(\text{NO ACE SPADES IN } n \text{ DRAWS})$

$= \frac{51}{52}\frac{50}{51}\frac{49}{50}$ $= \frac{52-n}{52}$ DEPENDENT DRAWINGS