Note Title 12/2/2009

STT 200 12-2-09

Read Chapter 20 : Testing Hypotheses About Proportions
Particular attention will be given to:

- Null and alternative Hypotheses pg. 509.
- SD(pHAT) calculated at a point p0 pg. 509.
- Test Statistic ~ z = (pHAT - p0) / SD(pHAT) pg.509.
- p-Value pg. 511 (not the usual p as used for population fraction).
- z-Test pg. 513.
- Other hypotheses pg. 515 (see "Alternative Alternatives).
- Summary beginning pg. 519.

NOTATION $H_1 = H_A$

ALT HYP

Lecture 12-2-09 will go over the following:

1. In a typical season one particular menu item accounts for around 17 percent of red meat orders, but a promotion has possibly increased that. A random sampling of 200 red meat orders from 16,000 orders during one week finds 46 for the item (rather **more than** the 34 expected if $p_0 = 0.17$ applies). It is desired to test the hypothesis H_0: p = 0.17 versus the alternative hypothesis H_A: $p > 0.17$.

a. Determine pHAT from this data.
$\hat{p}$ = 46/200 = .23

$\hat{p} = .23 > p_0 = .17$

b. Is this test one-sided or two-sided?
one-sided because H_A is entirely to one side of H_0

c. Determine SD(p_0).

text uses SD($\hat{p}$) but defines it as $\sqrt{\dfrac{p_0(1-p_0)}{n}} = \sqrt{\dfrac{.17 \times .83}{200}} = .0266$

FIRE ALARM: NULL HYPOTHESIS IS NO FIRE

ERROR OF TYPE I: FALSELY REJECT H_0

II: FAIL TO REJECT H_0 WHEN YOU SHOULD.

STATISTICAL TEST: IN PAST FOUND $p_0 = .17$ OF ORDERS FOR "RED MEAT."

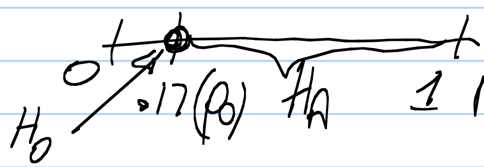
H_0: PRESENT RATE IS $p_0 = .17$

H_A: PRESENT RATE IS $> p_0$.

GATHER DATA IN FORM OF SAMPLE OF $n = 200$ FROM POP^N OF $16000 = N$.

WE FIND 46 ORDERS FOR RED MEAT.

$\hat{p}$ = SAMPLE FRACTION = 46/200 = .23



p = TRUE PRESENT PROPN

↑ LOOKS LIKE OUR EST'D SD OF $\hat{p}$ WHICH WAS USED IN CI AND IS $\sqrt{\hat{p}(1-\hat{p})}/\sqrt{n}$

DON'T USE THIS

INSTEAD TEXT $SD(\hat{p}) = \sqrt{p_0(1-p_0)}/\sqrt{n}$

NOTE: THIS IS NOT the SD of $\hat{p}$ as used in CI and estimated by $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.23 \times .77}{200}} = .0297$.

I USED $SD(p_0) = \sqrt{\frac{.17 \cdot .83}{200}}$ NOT $\sqrt{.23 \cdot .77/200}$ AS IN CI

d. Determine the value of the test statistic z.

$$z = \frac{\hat{p}-p_0}{SD(p_0)} = \frac{.23-.17}{.0266} = \mathbf{2.256} \quad \left(\text{NOT } \frac{.23-.17}{.0298} = 2.01\right)$$

$$z = \frac{\hat{p}-p_0}{SD(p_0)} = \frac{.23_{OBS} - .17_{H_0}}{\sqrt{.17 \cdot .83/200}}$$

2.256

HAS ≈ NORMAL [0, 1] OR Z DIST^N IF $p \sim .17$

e. Determine the p-value P(Z > test statistic value z from (d)) using the z-table.

P(Z > 2.256) ~ P(Z > 2.26) = 1 - 0.9881 = .0119 (table)

.9881

2.262

z	.06
2.2	**0.9881**

p-VALUE IS JUST $P(Z > 2.256) = 0.119$ (OBS z)

f. A statistical test of the hypothesis H_0: p = 0.17 versus the alternative hypothesis H_A: p > 0.17 will take the action of "rejecting the null hypothesis H_0: p = 0.17" if the p-value (e) is less than $\alpha = 0.01$. If not we say the test has failed to reject H_0: p = 0.1. Using p-value (e) what action is taken?

Fail to reject H_0 since p-value 0.0119 is not less than α = 0.01.

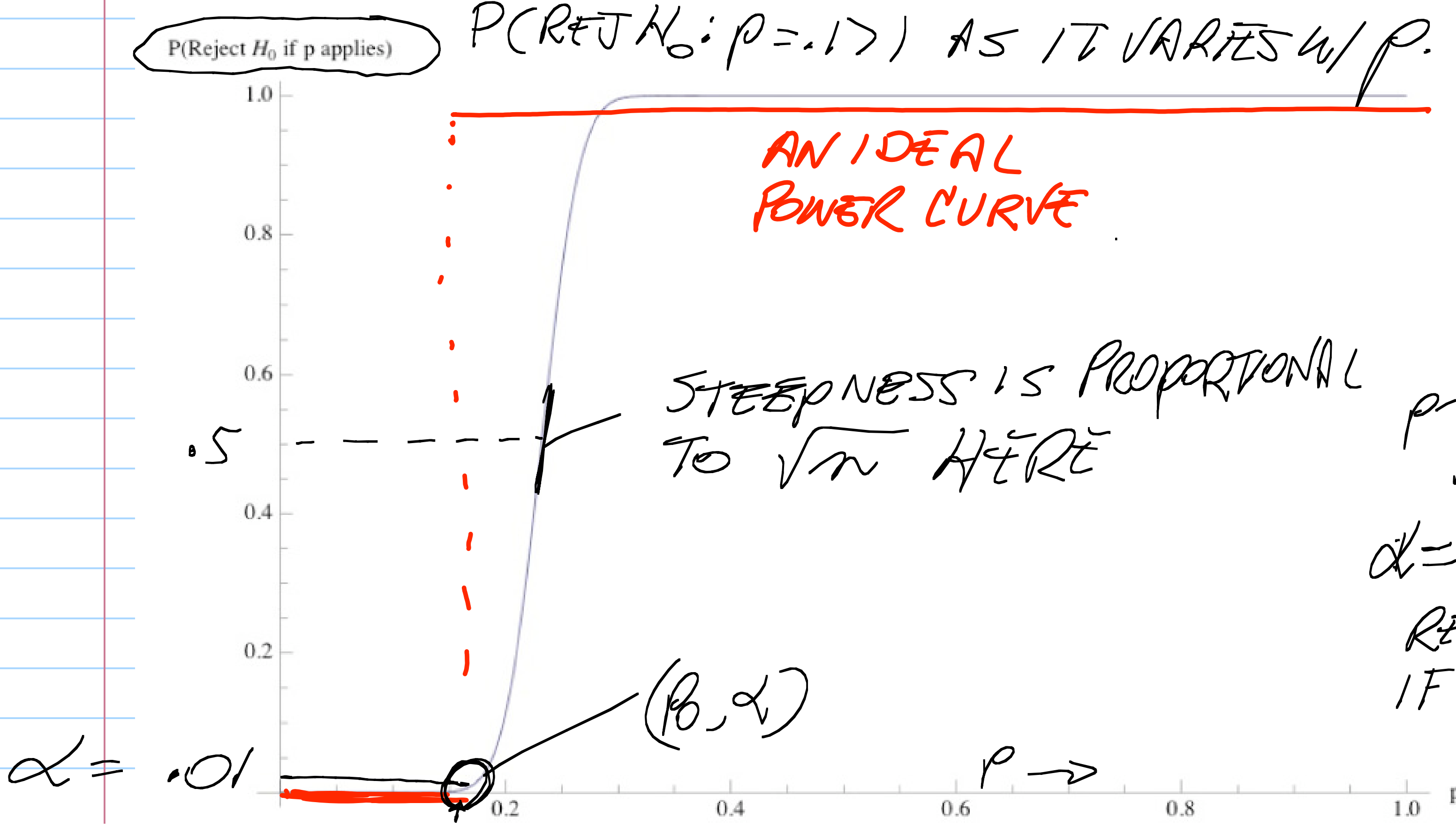
TEST: REJ H_0 IF p-VALUE < $\alpha = .01$

The value α called the "significance level of the test" is chosen by the experimenter. Its practical meaning is the probability of "error of the first kind" which is in turn equal to the probability that H_0: p = 0.17 will be falsely rejected when indeed p = 0.17 (the value p_0). This would be a "false rejection."

SUMMARY: p-VALUE IS PROB'Y OF GETTING MORE EVIDENCE AGAINST H_0 THAN YOU GOT IF H_0 IS TRUE.

TEST IS REJ H_0 IF p-VALUE $< \alpha = .01$ YOU CHOOSE.

g. Sketch the power curve of this test. Include α, p_0, in the sketch and also identify the roll of $\sqrt{n}$ (this is not in the readings, we will do it in class).



INGREDIENTS

$n = 200$

TEST STAT

$$z_{OBS} = \frac{\hat{p} - .17}{\sqrt{\frac{.17 \cdot .83}{200}}}$$

p-VALUE

$P(Z > z_{OBS})$

$\alpha = .01$

REJ $H_0 : p = .17$

IF p-VALUE $< .01 = \alpha$.

.17

$\uparrow p=0$ NO RED MEAT ORDERS EVER

$\uparrow p=1$ EVERYBODY WANTS RED MEAT

$P(Z > z_{OBS}) \sim 1 \not< .01$

FAIL TO REJECT H_0

$P(Z > z_{OBS}) \sim 0 < .01$

REJ H_0

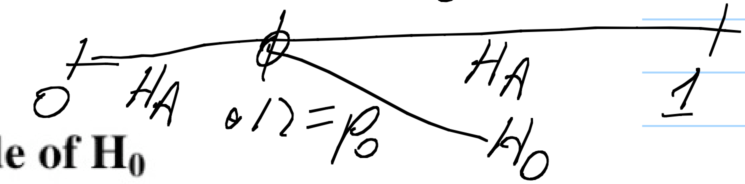
NOTE

2. In a typical season one particular menu item accounts for around 17 percent of red meat orders, but a promotion has possibly changed that. A random sampling of 200 red meat orders from 16,000 orders during one week finds 46 for the item (rather **different from** the 34 expected if p0 = 0.17 applies). It is desired to test the hypothesis H_0: p = 0.17 versus the alternative hypothesis H_A: $p \neq 0.17$.

a. Determine pHAT from this data.

$\hat{p}$ = 46/200 = .23 AS BEFORE

2-SIDED TEST.



b. Is this test one-sided or two-sided?

one-sided because H_A is NOT entirely to one side of H_0

c. Determine SD(p_0).

text uses SD($\hat{p}$) but defines it as $\sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.17 \times .83}{200}} = .0266$

AS BEFORE SAME $z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$

d. Determine the value of the test statistic z.

$z = \frac{\hat{p}-p_0}{SD(p_0)} = \frac{.23 - .17}{.0266} = 2.256$ (NOT $\frac{.23-.17}{.0298} = 2.01$)

e. Determine the p-value **P(| Z | > | test statistic value z from (d) |)** using the z-table.

ONE SIDED → **1 - .9881 = .0119**

2 (0.0119) ~ .024 two-sided test

TWO SIDED

↑ p-VALUE = .024

z	.06
2.2	0.9881

$z = 2.26$

BUT NOW p-VALUE

= P(DATA WORSE "THAN WE GOT)

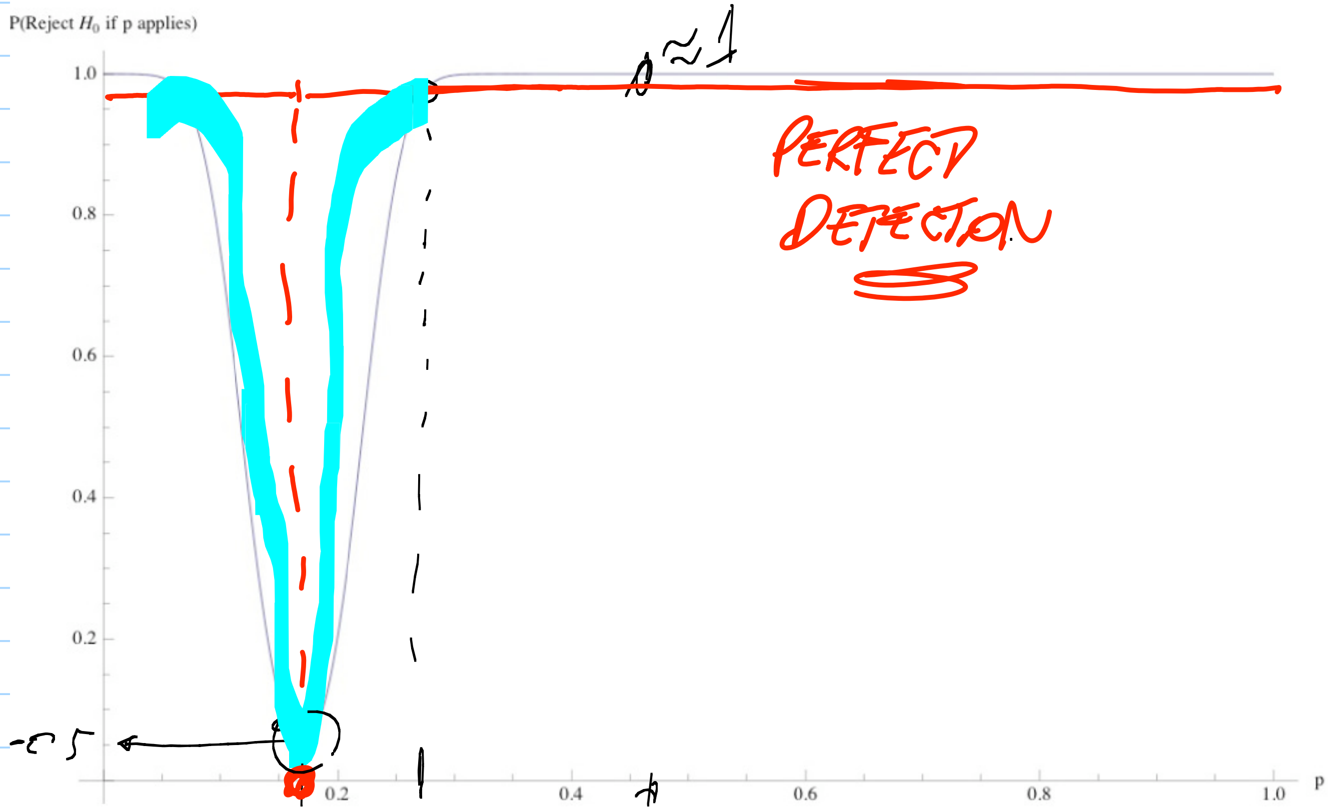
$P(|Z| > |z|) = P(Z > |z|) + P(Z < -|z|)$

f. A statistical test of the hypothesis H_0: p = 0.17 versus the alternative hypothesis H_A: p   0.17 will take the action of "rejecting the null hypothesis H_0: p = 0.17" if the p-value (e) is less than $\alpha = 0.05$. Using p-value (e) it this action taken? If not we say the test has failed to reject H_0: p = 0.17.

Reject H_0 since p-value 0.025 is less than $\alpha = 0.05$.

$\alpha = .05$ NOW (i FALSE REJ OF $H_0: p = .17$ OCCURS 5/100 TIMES IF $p = .17$)

g. Sketch the power curve of this test. Include α, p_0, in the sketch and also identify the roll of $\sqrt{n}$ (this is not in the readings, we will do it in class).



$\alpha = .05$

$n = 200$

FIND $\hat{p} = .23$

RECALL $p_0 = .17$

$H_0: p = .17$

$H_A: p \neq .17$

$p = .17$ $.27$ $P(\text{REJ}) \approx 1$

GIVEN H_0 H_A IS IT ONE-SIDED OR TWO SIDED?

GIVEN p-VALUE (SUMMARIZES EVIDENCE AGAINST H_0 OBTAINED FROM DATA — BUT SMALL p-VALUE INDICATES STRONG EVIDENCE AGAINST H_0)

AND GIVEN α

p-VALUE (SAY) .022

α = (SAY) .1 TEST REJ H_0 SINCE p-VALUE $.022 < \alpha = .1$

TECH $SD(p_0)$ (TEXT $SD(\hat{p})$) $= \sqrt{p_0(1-p_0)/n}$

z STATISTIC $z = \dfrac{\hat{p} - p_0}{SD(p_0)}$

p-VALUE = P(MORE EVIDENCE AGAINST H_0 THAN WE GOT)

a) $H_0: p = .17$ $H_A: p < .17$

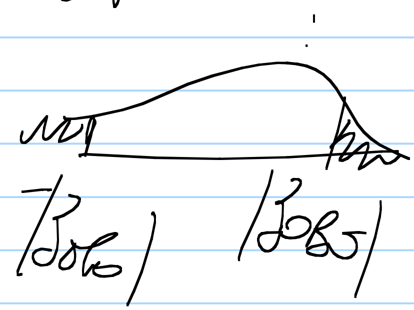
p-VALUE $= P(Z < z_{OBS})$

b) $H_0: p = .17$ $H_A: p > .17$

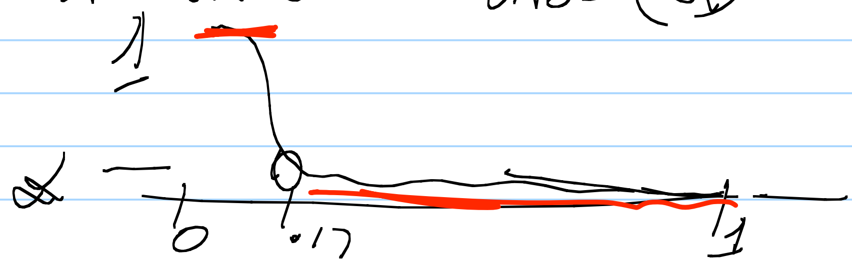
p-VALUE $P(Z > z_{OBS})$

c) $H_0: p = .17$ $H_A: p \neq .17$ TWO SIDED

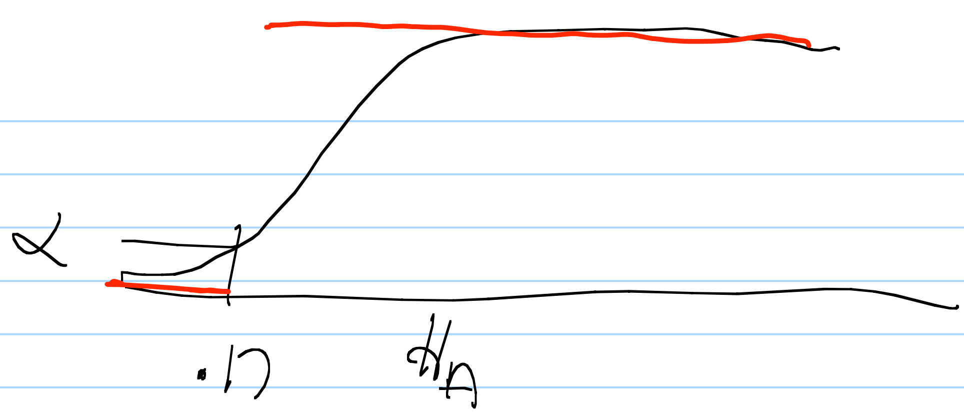
p-VALUE $= 2P(Z > |z_{OBS}|)$



SKETCH PWR CURVE CASE (C)



(b)



(c)