1. $X = \# \text{ of heads in 2 tosses}$

<table>
<thead>
<tr>
<th>outcomes</th>
<th>HH</th>
<th>HT</th>
<th>TH</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$p(x)$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

**Expected value of $X = E(X)$**

$$E(X) = \sum_{all \ x} x \cdot p(x)$$

$$= 2 \left( \frac{1}{4} \right) + 1 \left( \frac{1}{4} \right) + 1 \left( \frac{1}{4} \right) + 0 \left( \frac{1}{4} \right)$$

$$= 1$$

**Note:** this is not a probability, it means the expected value of $X$ is 1 (we expect to get 1 heads).

3. $X_1 = \# \text{ of heads on 1 coin toss}$

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<th>T</th>
</tr>
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<tr>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$p(x_1)$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

$$E(X_1) = \sum_{all \ x} x_1 \cdot p(x_1) = 1 \left( \frac{1}{2} \right) + 0 \left( \frac{1}{2} \right) = \frac{1}{2}$$

We expect $\frac{1}{2}$ heads. The values don't always make perfect logical sense.
4. \( X_1 \) = \# of heads on first coin toss in a total of 2 tosses

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<th>TT</th>
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</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( p(x_1) )</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
</tr>
</tbody>
</table>

\[
E(X_1) = \sum_{\text{all } x} x_1 p(x_1) = \left( \frac{1}{4} \right) + \left( \frac{1}{4} \right) + 0 \left( \frac{1}{4} \right) + 0 \left( \frac{1}{4} \right) = \frac{1}{2}
\]

5. Additivity Property of Expectations

\[
E(aX + bY + c) = aE(X) + bE(Y) + c
\]

\( a, b, c \) are constants, \( X, Y \) are random variables.

6. \[
\text{net} = 1.3X + 4 - (1.3)(2.8)X - .17Y + (1.3)(1.4) + 3.2
\]
\[
= [1.3 - (1.3)(2.8)]X + [-.17]Y + [(1.3)(1.4) + 3.2]
\]
\[
= 0.936X + 0.83Y + 5.02
\]

Find \( E(\text{net}) \). (Apply additive property)

\[
E(\text{net}) = E(0.936X + 0.83Y + 5.02)
\]
\[
= 0.936 E(X) + 0.83 E(Y) + 5.02
\]
\[
= 0.936 (23.69) + 0.83 (27.94) + 5.02
\]
\[
= 50.38404 \text{ million}
\]
7. Unbalanced coin:

\[ P(H) = 0.54, \quad P(T) = 1 - 0.54 = 0.46 \]

\[ X = \text{# of heads in 2 tosses} \]

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<td>x</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</table>

\[ p(x) = (0.54)(0.54), \quad (0.54)(0.46), \quad (0.46)(0.54), \quad (0.46)(0.46) \]

\[ E(X) = \sum_{\text{all } x} x p(x) = 2(0.54)^2 + 1(0.54)(0.46) + 1(0.46)(0.54) + 0(0.46)^2 \]

\[ = 1.08 \]

8. Same situation as Q7.

\[ X = \text{# of heads in 2 independent tosses} \]

| x | 2 | 1 | 0 |

\[ p(x) = 0.54^2, \quad (0.54)(0.46) + (0.46)(0.54), \quad 0.46^2 \]

\[ \text{there are 2 ways to get 1 Head: HT and TH} \]

\[ E(X) = \sum_{\text{all } x} x p(x) = 2(0.54)^2 + 1[(0.54)(0.46) + (0.46)(0.54)] + 0(0.46)^2 \]

\[ = 1.08 \]

Q8 gets the same result as Q7, the difference is in the setup of the problem.
9. \( X_i = \# \) of heads in 1st toss

\[ x_i : 1 \quad 0 \]
\[ p(x_i) \quad .54 \quad .46 \]

\[ E(X_1) = 1(.54) + 0(.46) = .54 \]

10. Setup the problem as follows (to use additivity property):

\( X_1 = \# \) of heads in 1st toss

\( X_2 = \# \) of heads in 2nd toss

\vdots

\( X_{100} = \# \) of heads in 100th toss

\[ E(X_1) = E(X_2) = \ldots = E(X_{100}) = .54 \] (result from Q9)

Want to find: \( E(\text{net}) \)

\[ \text{net} = X_1 + X_2 + \ldots + X_{100} \]

\[ E(\text{net}) = E(X_1 + X_2 + \ldots + X_{100}) \]
\[ = E(X_1) + E(X_2) + \ldots + E(X_{100}) \]
\[ = .54 + .54 + \ldots + .54 \]
\[ = .54(100) \]
\[ = 54 \]

In 100 tosses, we expect 54 heads.
More Important Properties:

1. If $X$ & $Y$ are "Independent",
   \[ E(XY) = E(X)E(Y) \]

Variance of $X$:
\[ \text{Var}(X) = E(X^2) - (E(X))^2 \]

SD of $X$:
\[ \text{SD}(X) = \sqrt{\text{Var}(X)} \]

11. Unbalanced coin:
   \[ P(H) = 0.54, \quad P(T) = 0.46 \]

   $X_i$ = # of heads in $i$ tosses

   \[
   \begin{array}{c|c|c}
   x_i & 1 & 0 \\
   \hline
   p(x_i) & 0.54 & 0.46 \\
   \end{array}
   \]

   \[
   E(X_i) = \sum_{x_i} x_i p(x_i) = 1(0.54) + 0(0.46) = 0.54
   \]

12. Same setup as Q11, but want to find $E(X_i^2)$.

   \[
   E(X_i^2) = \sum_{x_i} x_i^2 p(x_i) = 1^2(0.54) + 0^2(0.46) = 0.54
   \]
13. Find $\text{Var}(X_1)$. (Use results from Q11 & Q12)

$$\text{Var}(X_1) = \text{E}(X_1^2) - \left(\text{E}(X_1)\right)^2$$

$$= .54 - .54^2$$

$$= 0.2484$$

14. Same as Q10.

$\text{E}(\text{net}) = 54$

15. The coin is tossed 100 times. Each toss is independent.

$\text{net} = X_1 + X_2 + \cdots + X_{100}$

Find $\text{Var}(\text{net})$.

**Property of Variance**: If $X$ and $Y$ are independent

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

$\text{Var}(X_1) = \text{Var}(X_2) = \cdots = \text{Var}(X_{100}) = 0.2484$ (from Q13)

$\text{Var}(\text{net}) = \text{Var}(X_1 + X_2 + \cdots + X_{100})$

$$= \text{Var}(X_1) + \text{Var}(X_2) + \cdots + \text{Var}(X_{100}) \quad \text{(independence)}$$

$$= 0.2484 + 0.2484 + \cdots + 0.2484$$

$$= 0.2484 (100)$$

$$= 24.84$$
16. \( E(X) = .54 \)
   \( \text{Var}(X) = 24.84 \)
   \( SD(X) = 4.984 \)

68\% interval: \((49.016, 58.984)\)