1. $0.49(10000) = 4900$

2. $0.60(1000) = 600$

3. I will use the notation $(\#, \#)$.

   \[
   \text{all possible outcomes} = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\
   (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\
   (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\
   (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\
   (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\
   (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}
   \]

   Note: this is with replacement.

   There are 36 outcomes in total from counting the above.

   - $P(R+G = 7) = \frac{6}{36} = \frac{1}{6}$ because for $R+G = 7$, we can have the following outcomes:
     - $(1,6)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,2)$, $(6,1)$

   which are 6, and we divide 6 by the total number of outcomes to find the probability.

   a) $P(R > G + 2) = \frac{6}{36} = \frac{1}{6}$

   Outcomes: $(4,1), (5,1), (5,2), (6,1), (6,2), (6,3)$

   b) $P(R - G = 4) = \frac{2}{36} = \frac{1}{18}$

   Outcomes: $(5,1), (6,2)$

   c) $P(R^2 = 25) = \frac{6}{36} = \frac{1}{6}$

   Outcomes: $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
4. \( P(\text{Jack gets/draws a$5 bill}) = \frac{1}{4} \)

Outcome: $5 bill

There are 4 bills in total.

5. a) Depends on what Jack got, since he draws first.
   b) 0
   c) \( \frac{1}{3} \)

6. all possible outcomes = \( \{ (1a, 1b), (1a, 1c), (1a, 5), (1b, 1a), (1b, 1c), (1b, 5), (1c, 1a), (1c, 1b), (1c, 5), (5, 1a), (5, 1b), (5, 1c) \} \)

   Note: this is without replacement

7. a) \( \frac{3}{12} = \frac{1}{4} \)
   b) \( \frac{3}{9} = \frac{1}{3} = 0 \)
   c) \( \frac{3}{9} = \frac{1}{3} \)

   Note: in (b) we restrict ourselves to looking at just the outcomes in which Jack gets 5.
   Smaller group: \( (5, 1a), (5, 1b), (5, 1c) \)

   and we determine how many of these 3 outcomes are ones in which Jill gets 5 (this has to do with conditional probability)

   Note: in (c) we restrict ourselves to looking at the outcomes:
   \( (1a, 1b), (1a, 1c), (1a, 5), (1b, 1a), (1b, 1c), (1b, 5), (1c, 1a), (1c, 1b), (1c, 5) \)
8. Parts (b) and (c) in Q5 and Q7 are the same.

9. a) \( P(\text{first ball is red}) = \frac{2}{6} = \frac{1}{3} \)
   b) \( P(\text{first ball is red OR green}) = \frac{5}{6} \)
   c) Depends on what color ball was drawn on the first draw.
      * answer = \( \frac{1}{3} \) (explanation on last page)

10. I'm going to label the contents \([\text{ra},\text{rb},\text{ga},\text{gb},\text{gc},\text{y}]\).
    
    **all possible outcomes** = \(\{(\text{ra},\text{rb}), (\text{ra},\text{ga}), (\text{ra},\text{gb}), (\text{ra},\text{gc}), (\text{ra},\text{y})
        
        (\text{rb},\text{ra}), (\text{rb},\text{ga}), (\text{rb},\text{gb}), (\text{rb},\text{gc}), (\text{rb},\text{y})
        
        (\text{ga},\text{ra}), (\text{ga},\text{rb}), (\text{ga},\text{gb}), (\text{ga},\text{gc}), (\text{ga},\text{y})
        
        (\text{gb},\text{ra}), (\text{gb},\text{rb}), (\text{gb},\text{ga}), (\text{gb},\text{gc}), (\text{gb},\text{y})
        
        (\text{gc},\text{ra}), (\text{gc},\text{rb}), (\text{gc},\text{ga}), (\text{gc},\text{gb}), (\text{gc},\text{y})
        
        (\text{y},\text{ra}), (\text{y},\text{rb}), (\text{y},\text{ga}), (\text{y},\text{gb}), (\text{y},\text{gc})\} \)

    **Note:** this is without replacement

    \( P(\text{Second draw is red}) = \frac{10}{30} = \frac{1}{3} \) (Same as Q9(c))

    **Outcomes:** \((\text{ra},\text{rb}), (\text{rb},\text{ra}), (\text{ga},\text{ra}), (\text{ga},\text{rb}), (\text{gb},\text{ra})
      
      (\text{gb},\text{rb}), (\text{gc},\text{ra}), (\text{qc},\text{rb}), (\text{y},\text{ra}), (\text{y},\text{rb})\)

    The setup is the same as in Q9(c) so we expect to get the same answer.

11. a) restricted to looking at outcomes:
    
    \((\text{ra},\text{rb}), (\text{ra},\text{ga}), (\text{ra},\text{gb}), (\text{ra},\text{gc}), (\text{ra},\text{y})
      
      (\text{rb},\text{ra}), (\text{rb},\text{ga}), (\text{rb},\text{gb}), (\text{rb},\text{gc}), (\text{rb},\text{y})\)

    \# of outcomes with 2\text{nd} draw being red = 2

    \( P(\text{2nd draw is red if 1st draw was red}) = \frac{2}{10} = \frac{1}{5} \)

    \( = P(\text{2nd draw red} \mid \text{1st draw red}) \)
b) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw not red}) = \frac{9}{20} = \frac{2}{5} \)

c) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw yellow}) = \frac{2}{15} = \frac{2}{5} \)

d) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw green}) = \frac{6}{15} = \frac{2}{5} \)

e) \( P(2^{nd} \text{ draw yellow} \mid 1^{st} \text{ draw yellow}) = \frac{9}{5} = 0 \)

12. \( \text{all possible outcomes} = \{ (ra, ra), (ra, rb), (ra, qa), (ra, qb), (ra, gc), (ra, gray), (rb, ra), (rb, rb), (rb, qa), (rb, qb), (rb, gc), (rb, gray), (qa, ra), (qa, rb), (qa, qa), (qa, gb), (qa, gc), (qa, gray), (qb, ra), (qb, rb), (qb, qa), (qb, gb), (qb, gc), (qb, gray), (qc, ra), (qc, rb), (qc, qa), (qc, gb), (qc, gc), (qc, gray), (y, ra), (y, rb), (y, qa), (y, gb), (y, gc), (y, gray) \} \)

Note: this is without replacement

\( P(2^{nd} \text{ draw is red}) = \frac{12}{26} = \frac{1}{3} \)

Outcomes: (ra, ra), (ra, rb), (rb, ra), (rb, rb), (qa, ra), (qa, rb),
(qb, ra), (qb, rb), (gc, ra), (gc, rb), (y, ra), (y, rb)

We get the same answer as Q9(c), but the setup is different. This will not always be true.

13. a) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw red}) = \frac{4}{12} = \frac{1}{3} \)

b) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw not red}) = \frac{8}{24} = \frac{1}{3} \)

c) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw yellow}) = \frac{3}{6} = \frac{1}{3} \)

d) \( P(2^{nd} \text{ draw red} \mid 1^{st} \text{ draw green}) = \frac{6}{18} = \frac{1}{3} \)

e) \( P(2^{nd} \text{ draw yellow} \mid 1^{st} \text{ draw yellow}) = \frac{1}{6} \)
Venn Diagrams

1. The smaller circles represent events $A$ and $B$ respectively. We can draw events $A$ and $B$ inside a box or a circle, both are okay. ($A$ and $B$ drawn inside a circle on worksheet)

$A \cup B$: reads "A union B" or "A OR B"

$A \cap B$: reads "A intersect B" or "A AND B"
b) All non-overlapping areas always add up to 1.

\[ P(B | A) = \frac{P(B \cap A)}{P(A)} = \frac{0.4}{0.7} = \frac{4}{7} \]

We restrict ourselves to the circle A, and find the portion of B relative to this restricted area.

d) \[ P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.6} = \frac{2}{3} \]

We restrict ourselves to the circle B, and find the percentage of A relative to this restricted area.

e) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ = 0.7 + 0.6 - 0.4 \]
\[ = 0.9 \]

f) Same as (e).

9. (c) There are 2 cases to consider.

Case 1
The 1st ball drawn was red.
\[ P(2\text{nd ball is red}) = \]
\[ \text{chance that 1st ball is red} \times \text{chance that 2nd ball is red} \]
\[ = \frac{2}{6} \times \frac{1}{5} = \frac{2}{30} \text{ (1 red left on 2nd draw)} \]
Case 2
The first ball drawn is not red.

\[ P(\text{2nd ball is red}) = \text{chance that 1st ball is not red} \times \text{chance that 2nd ball is red} \]
\[ = \frac{4}{5} \times \frac{2}{5} \]
\[ = \frac{8}{25} \quad (2 \text{ red balls left in 2nd draw}) \]

\[ P(\text{2nd ball is red}) = \text{case 1} + \text{case 2} \quad \text{(all possible cases added together)} \]
\[ = \frac{2}{15} + \frac{8}{25} = \frac{20}{75} \]
\[ = \frac{1}{3} \]

*You will better understand this question if you look at it last.*