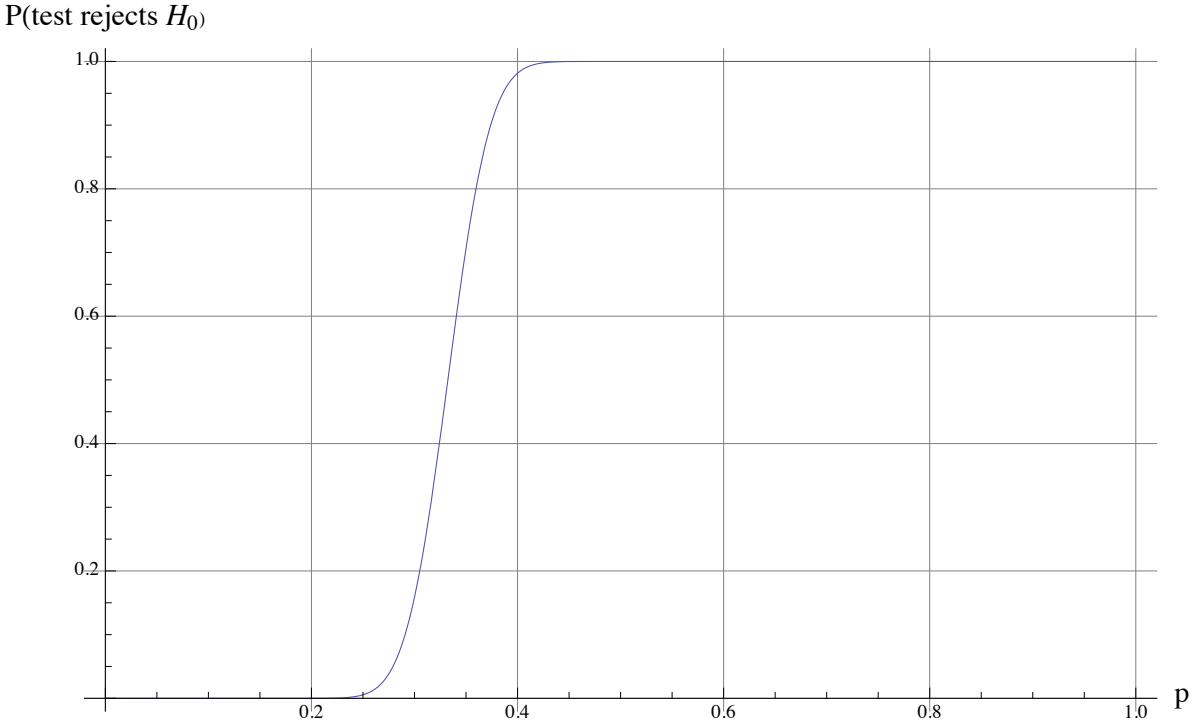
Recitation Assignment due 12-8-09.

Exam 5 coverage: Most of the exam questions go to material from chapters 16 and 20, but some may touch on supporting material from chapter 15. Expect to see questions relating to and couched in language developed in lectures and recitation assignments.

1. Here is the power function for a particular test about p = fraction of beverage orders for Coke in a restaurant. Suppose that $p_0 = 0.3$.



a. Determine the null hypothesis H_0 and the alternative hypothesis H_A.

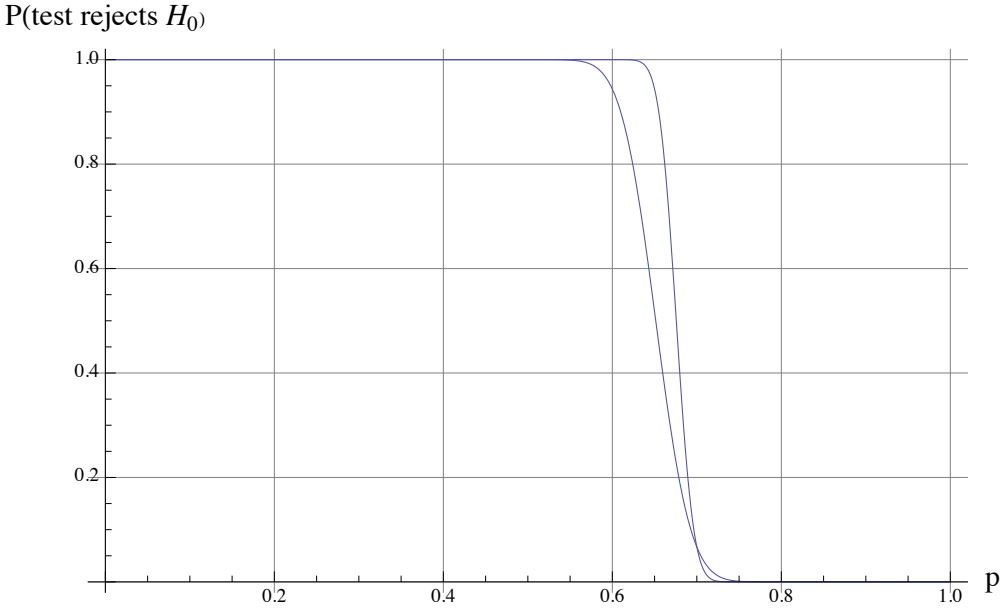
b. Determine the probability of type one error α (i.e. chance the test will reject the null hypotheses if $p = p_0$). This α is specified by the experimenter constructing the test and is termed the "significance level of the test." One also speaks of the "α level of the test."

c. Determine the probability of type two error (chance the test will *fail to reject* H_0) if p = 0.35.

d. If Coke sales are running at 10% is there much chance of rejecting the null hypothesis using the test with this power curve?

e. If Coke sales are running at 40% is there much chance of rejecting the null hypothesis using the test with this power curve?

2. Two power curves are overlaid below. Both have the same significance level α and the same null and alternative hypotheses.



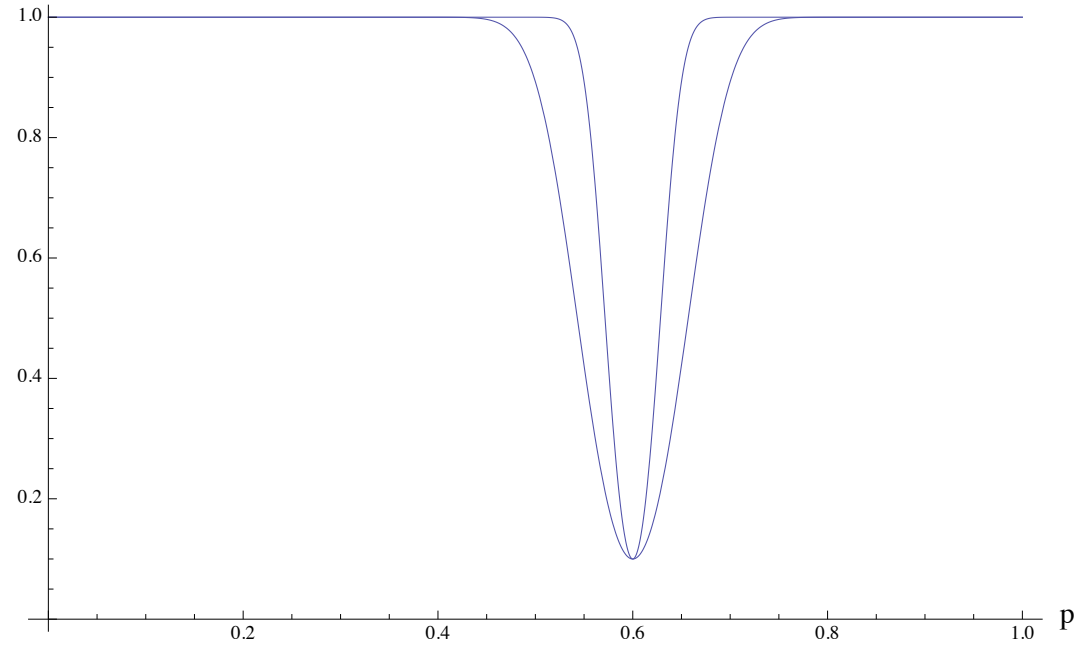
a. Identify the null and alternative hypotheses, α. Which test is better?

b. Is this a one sided or two sided test?.

c. By hand, draw in the *ideal power curve* based on *perfect information*.

3. Two power curves are overlaid below. Both have the same significance level α and the same null and alternative hypotheses.

P(test rejects H_0)



a. Identify the null and alternative hypotheses. Is the test one or two sided?

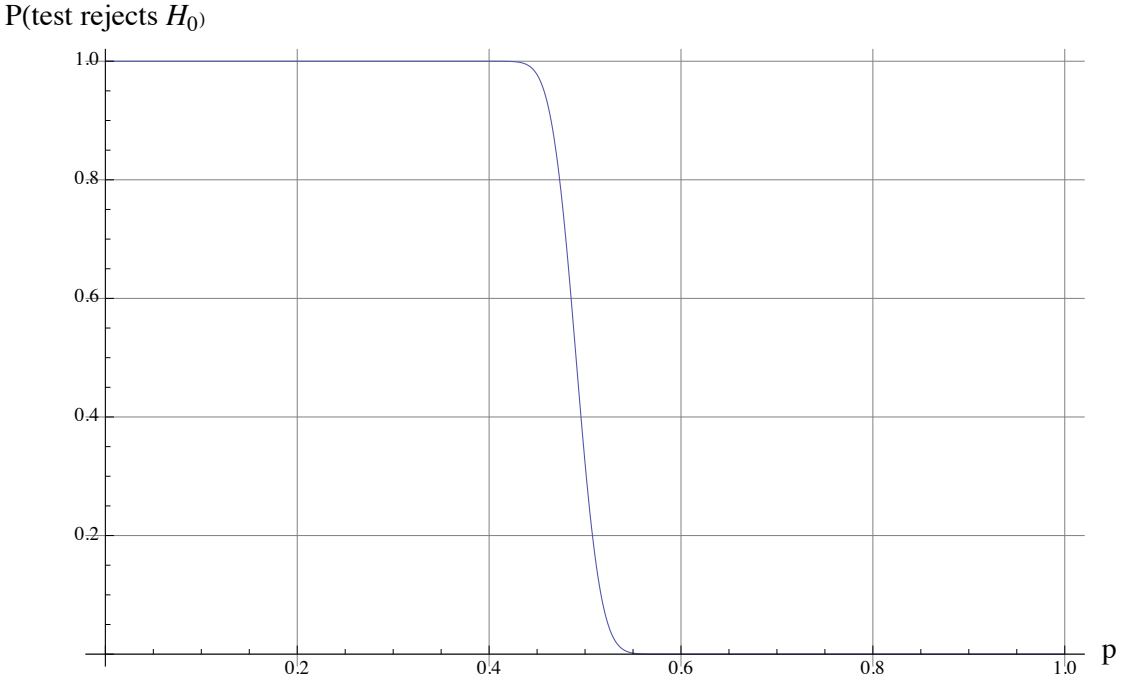
b. Determine the significance level α.

c. By hand, draw in the *ideal power curve* based on *perfect information*.

d. Which is the power curve of the better test?

e. If p = 0.55 what is the probability of rejecting the null hypothesis uner each of the two tests?

4. An election pits Republican candidate against a Democrat candidate. Denote by p the fraction of the electorate currently favoring the Republican. Here is the power curve for a possible test (based on a random sample of 800 voters) of the ***null hypothesis that the Republican candidate is leading in votes***. The alternative hypothesis is the Republican candidate is not leading in votes.



a. Identify the null and alternative hypotheses on the p axis and label the p axis also as "fraction favoring Republican."

b. What is the value of the probability α of type one error (concluding that the Democrat is ahead when actually the Republican is at 50%)?

c. Which party should complain they would be treated unfairly by the test?

d. If the Republican share of the vote is p = 0.45 what is the probability that the test will reject the null hypothesis?

e. If you could devise a test with the same shape of power curve what would you do to the curve to make the associated test fair to both parties?

5. A statistical test of the null hypothesis that wine orders are running at 30% versus the alternative that they run at greater than 30% will be based on a random sample of 400 orders and α = 0.2. In the sample there are 150 wine orders.

a. With the given information what is the value of $\hat{p}$?

b. With the given information what is the value of p_0?

c. Determine the value of $SD(p_0) = \sqrt{\frac{p_0(1-p_0)}{n}}$. $SD(\hat{p})$ is the notation used for this in your textbook but let's not confuse it with the estimated SD of $\hat{p}$ (as used in CI) which is $\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$.

d. Determine the value of the ***test statistic*** denoted $z = \frac{\hat{p}-p_0}{SD(p_0)}$.

e. Determine the p-value = $P(Z > z)$ where z is the value of the test statistic (d).

f. The test rejects H_0 if p-value < α. What action is taken by the test? Is it to *reject the null hypothesis* or to *fail to reject the null hypothesis*?

g. Had there been 100 wine orders in the sample would there have been any reason to carry out the hypothesis test?

6. A statistical test of the null hypothesis that wine orders are running at 30% versus the alternative that they run at some value OTHER THAN 30% will be based on a random sample of 400 orders and $\alpha = 0.2$. In the sample there are **90** wine orders.

a. With the given information what is the value of $\hat{p}$?

b. With the given information what is the value of p_0?

c. Determine the value of $SD(p_0) = \sqrt{\frac{p_0(1-p_0)}{n}}$.

d. Determine the value of the ***test statistic*** denoted $z = \frac{\hat{p}-p_0}{SD(p_0)}$.

e. Determine the p-value = **2** P(Z > | z |) where z is the value of the test statistic (d). The reason for the **2** is that this two sided test rejects when $\hat{p}$ is either too large or too small so both tail probabilities have to be included.

f. What action is taken by the test? Is it to *reject the null hypothesis* or to *fail to reject the null hypothesis*?

g. Had there been 100 wine orders in the sample would there have been any reason to carry out the hypothesis test?