Solutions to Recitation 9-29-09

For Monday, 9-28-09 read from chapter 6 pp. 121-136. I plan to cover this material.

Your recitation assignment for 9-29-09 is to answer the following questions:

1. What are the mean and the standard deviation of the normal distribution written N(39, 16)?

2. IQ is distributed normal with mean 100 and standard deviation 15. If your IQ is 113 what is your standard score?

3. Refer to (2). What fraction of IQ scores fall in the range 85 to 100? DRAW THE PICTURE AND SHADE THE AREA.

4. Same as (3) but the range 70 to 130.

5. Same as (3) but the range 130 to infinity.

6. Same as (3) but the range 85 to 100.

7. Same as (3) but the range 115 to 130.

8. The standard normal Z has mean 0 and standard deviation 1. DRAW THE PICTURE. To determine the chance of $Z < 1.48$ (for example) we use a TABLE Z on page A-97. The number 1.48 is broken down into pieces 1.4 and .08 and we look up $P(Z < 1.48)$ as follows:

z	.08
1.4	$P(Z < 1.48)$

DO THIS and find the probability (that is, the fraction of the standard normal below 1.48). You will find an answer close to 0.93. What is it to table accuracy?

9. Refer to (8). Find $P(Z < 0.00)$. You should get 0.5. DRAW THE PICTURE!

10. Refer to (8). Find $P(Z < 1.00)$. Recalling the rule of thumb you should get $P(Z < 1.00) = \mathbf{0.5} + 0.68 / 2 = \mathbf{0.5} + 0.34 = \mathbf{0.84}$. Draw the picture! Check it out!

You are using $P(Z < 1.00) = P(Z < 0.00) + P(0 < Z < 1.00) = 0.5 + 0.34$.

The "0.5 +" portion was left off my original message to you.

11. What is the probability that a person has IQ < 121? All normals are alike in standard deviation units from their mean. That is

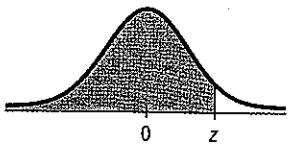
$P(IQ < 121) = P(Z <$ standard score of IQ **121**$) = P(Z < (121-100)/15)$

$= P(Z < 1.40)$.

Use the table to get this.

12. Refer to (11). What is $P(IQ > 121)$? Since $P(IQ = 121) = 0$ in a continuous model such as the normal, $P(IQ > 121) = 1 - P(IQ < 121)$. DRAW THE PICTURE AND SHADE THE RELEVANT AREA. Give the answer.

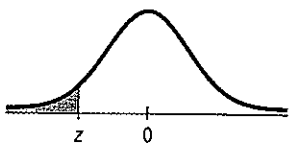
Table Z (cont.)
Areas under the standard Normal curve



	Second decimal place in z									
z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
		Q10								
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
	Q11							Q8		
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000†									

†For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

Table Z
Areas under the standard Normal curve



Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	*z*
									0.0000†	*−3.9*
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	*−3.8*
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	*−3.7*
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	*−3.6*
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	*−3.5*
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	*−3.4*
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	*−3.3*
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	*−3.2*
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	*−3.1*
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	*−3.0*
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	*−2.9*
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	*−2.8*
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	*−2.7*
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	*−2.6*
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	*−2.5*
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	*−2.4*
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	*−2.3*
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	*−2.2*
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	*−2.1*
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	*−2.0*
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	*−1.9*
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	*−1.8*
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	*−1.7*
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	*−1.6*
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	*−1.5*
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	*−1.4*
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	*−1.3*
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	*−1.2*
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	*−1.1*
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	*−1.0*
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	*−0.9*
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	*−0.8*
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	*−0.7*
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	*−0.6*
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	*−0.5*
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	*−0.4*
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	*−0.3*
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	*−0.2*
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	*−0.1*
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	*−0.0*

†For $z \leq -3.90$, the areas are 0.0000 to four decimal places.

Solutions to Recitation 9-29-09

1. $N(\underline{39}, \underline{16})$

↑ mean = μ ↖ entire second parameter = variance = σ^2

We want to write it in standard notation:

$N(39, 16) = N(39, \underline{4^2})$

The number before we square it is the standard deviation.

standard deviation = 4

2. The question wants us to convert the score of 113 to a "standard normal distribution" value (z-value).

We use the equation:

$$\frac{X - \mu}{\sigma} = z\text{-value}$$

where: X = the value you want to convert

μ = mean of the normal distr. that X comes from

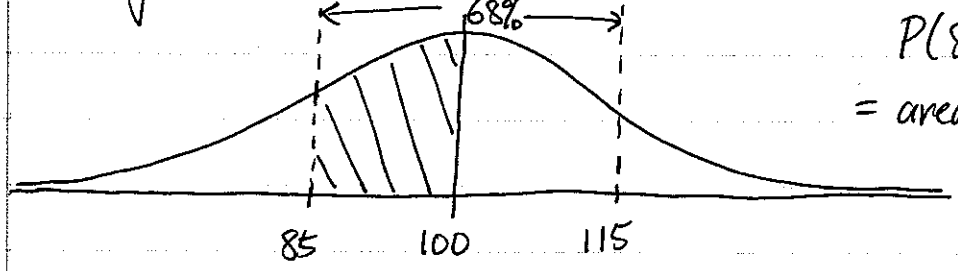
σ = SD " " " " " " " "

$$z\text{-value} = \frac{113 - 100}{15} = \frac{13}{15}$$

Questions 3-7 is for the normal distr. $N(100, 15^2)$.

3. Find $P(85 < IQ < 100)$.

- always draw the area 1st



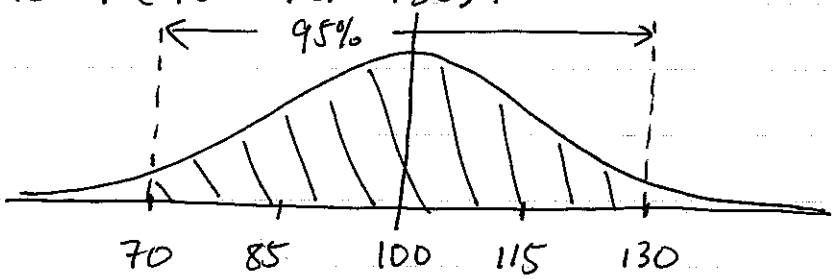
*remember $P(\ldots)$ = probability of what's inside brackets, is the same as area under the curve.

$P(85 < IQ < 100)$

$= \text{area} = \frac{68\%}{2} = 34\% = 0.34$

(using 68% area principle)

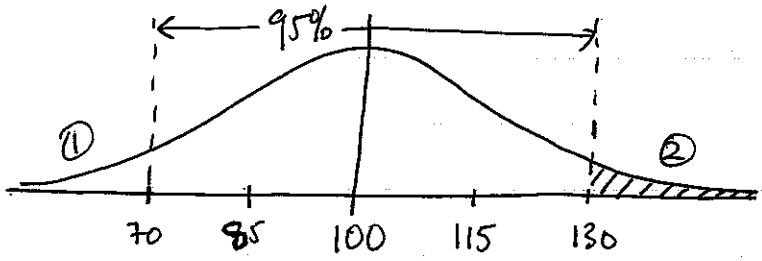
4. Find $P(70 < IQ < 130)$.



area $= P(70 < IQ < 130) = 95\%$ (by 95% area principle)

$= 0.95$

5. Find $P(130 < IQ < \infty) = P(IQ > 130)$.



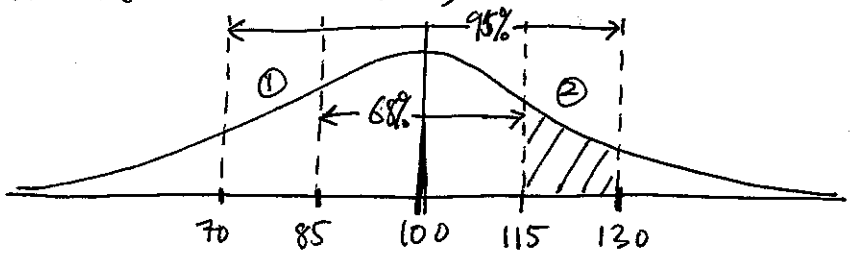
① + ② $= 100\% - 95\% = 5\%$

$\Rightarrow$ ② $= \frac{5\%}{2} = 2.5\%$ (because the normal distr. is symmetric)

$P(IQ > 130) = 2.5\% = 0.025$

6. Same as Q.3.

7. Find $P(115 < IQ < 130)$.



$95\% - 68\% =$ ① + ② $= 27\%$

② $= \frac{27\%}{2} = 13.5\%$

$P(115 < IQ < 130) = 13.5\% = 0.135$

*Note: In Q.3-7, we had nice numbers to work with (multiples of the standard deviation 15). When we do not have nice numbers, we can't use the 68% or 95% area principles. Instead we look up the areas in the standard normal distribution table (z-table).

8. Find $P(z < 1.48)$.

$P(z < 1.48) = 0.9306$ from z-table

$= 93.06\%$

*Note: to use the z-table, you must put the probability in the form:

$P(z \leq$ "some value")

because in the z-table, they shade from the left up to "some value" (must have "$\leq$").

9. Find $P(z < 0)$.

$P(z < 0) = 0.5 = 50\%$ from z-table (or from visualizing that the area is half the normal curve)

10. Find $P(z < 1)$.

$P(z < 1) = 0.8413 = 84.13\%$ from z-table

*Note: In Q.11-12, we do not have nice numbers to work with (what I mentioned earlier). So in order to find the areas, we need to look them up in the z-table. But we are not working with the "standard normal distribution" here, so first, we need to convert these numbers to z-values (we standardize).

(distributed)

11. IQ scores follow $N(100, 15^2)$.
Find $P(IQ < 121)$.

● we standardize: $\frac{X-\mu}{\sigma} = z\text{-value}$

$$P(IQ<121) = P(z < \frac{121-100}{15})$$
$$= P(z < 1.4)$$
$$= 0.9192 \quad = 91.92\%$$

12. Find $P(IQ > 121)$.
we standardize:

$$P(IQ > 121) = P(z > \frac{121-100}{15})$$
$$= P(z > 1.4)$$
$$= 1 - P(z \leq 1.4)$$

$P(z \leq 1.4)$ is in the format we want to look it up in the z-table
$= 0.9192$

* Note: for continuous distributions (normal distr. is continuous),
$\leq$ is the same as $<$ and
$\geq$ is the same as $>$

$$P(IQ > 121) = 1 - P(z \leq 1.4)$$
$$= 1 - 0.9192$$
$$= 0.0808 = 8.08\%$$

* if you draw out $P(z > 1.4)$, this is just the opposite area of Q11.

* Final Note: if you are stuck, draw out the area you want to find.