1. \( P(A) = 0.7, \ P(B) = 0.4, \ P(AB) = 0.2 \).

a. Venn diagram. **Hint:** \( P(A^c B) = P(A) - P(AB) \)

b. Tree diagram. **Hint:** \( P(B \mid A) = \frac{P(AB)}{P(A)} \)

\[
\begin{align*}
P(B) &= P(AB) + P(A^c B) = 0.2 + 0.2 = 0.4 \\
&= 1
\end{align*}
\]
1. \( P(A) = 0.7, P(B) = 0.4, P(AB) = 0.2. \)

c. From the Venn Diagram, find \( P(B) \) and \( P(A \mid B) \).
\[
P(B) = P(AB) + P(A^C B), \quad P(A \mid B) = \frac{P(AB)}{P(B)}
\]

\[
P(B) = P(AB) + P(A^C B) = 0.2 + 0.2 = 0.4
\]

(Bayes) \( P(A \mid B) \) \( \overset{\text{def}}{=} \frac{P(AB)}{P(B)} = \frac{0.2}{0.4} \frac{\text{FROM}}{\text{GIVEN}} \)

d. From the Tree Diagram determine \( P(A \mid B) \) (Bayes).
\[
P(A \mid B) = P(AB) / P(B)
\]

\[
\text{Tree} = \frac{0.2}{0.2 + 0.2}
\]

\[
P(A \mid B) = \frac{P(AB)}{P(B)} \quad \text{CONDITIONING EVENT-}
\]

\[
\text{Tree} = \frac{0.2}{0.4}
\]
2. $P(\text{OIL}) = 0.1$, $P(+ | \text{OIL}) = 0.7$, $P(+ | \text{no OIL}) = 0.2$.  

a. Tree. 

b. $P(\text{OIL} | +) \overset{\text{BAYES}}{=} \frac{P(\text{OIL} | +) \cdot P(\text{OIL})}{P(+)} = \frac{0.7 \cdot 0.1}{0.1 \cdot 0.7 + 0.9 \cdot 0.2} = 0.98$ 

c. Costs: test = 20, drill = 60. Gross from oil = 400. 

$E(\text{NET return from "just drill"}) = \sum \text{VALUE} \cdot \text{PR} = \sum x \cdot P(x) = 34 \cdot 0.1 + 54 \cdot 0.9 = -20$ 

Policy II

d. $E(\text{NET from "test, drill if +"}) = 0.7 \cdot 320 + 1.3 \cdot (-20) + 0.9 \cdot 2 \cdot (-80) + 0.9 \cdot 0.8 \cdot (-20) = 22.4$ 

TOTAL IS $E(\text{NET Policy II})$.
There was no question 3.

a. What is the approximate probability of landing on Boardwalk (or any other property) in Monopoly?

\[ P \approx \frac{1}{2} \]

b. If the rent on that property is $200 what is the expected return to the owner from one player-circuit of the board?

\[ \frac{200}{1} + \frac{0}{1} + \frac{1}{1} \]

c. If a player owns properties with rents $100, $150, $300 what is the expected return from three player-circuits of the board?

\[ \frac{100}{1} + \frac{150}{1} + \frac{300}{1} \]
4. \( P(A) = 0.4, P(B) = 0.5, P(AB) = 0.20 \).

\begin{align*}
\text{a.} \quad & P(A \cup B) \\
& P(A \cup B) = P(A) + P(B) - P(AB) \text{ always.} \\
& = 0.4 + 0.5 - 0.2 = 0.7
\end{align*}

\begin{align*}
\text{b.} \quad & \text{From definition } P(B \mid A). \\
& P(B \mid A) = \frac{P(AB)}{P(A)}. \\
& = \frac{0.2}{0.4} = \frac{1}{2}
\end{align*}

c. Are A, B independent of each other? Show reasoning! Does \( P(AB) = P(A)P(B) \)?

\begin{align*}
\text{Yes, } P(B) \neq P(B \mid A) \\
\therefore & 0.5 \neq \frac{1}{2} \text{, so } \\
\therefore & \text{INDEP}
\end{align*}
5. $P(A) = 0.4$, $P(B) = 0.3$, $P(B \mid A) = 0.6$.

a. Give $P(AB)$.
   
   $P(AB) = P(A) \cdot P(B \mid A)$ always if $P(A) > 0$.

b. Are $A$, $B$ independent? Is $P(B) = P(B \mid A)$?

c. Fill out a complete Venn Diagram.
6. X = draw from \{2, 4, 4, 6\}. Y draw from \{2, 2, 2, 6\}.

a. \[ E X = \frac{2 + 4 + 4 + 6}{4} = \frac{16}{4} = 4 \]
\[ E X = 4 \] 
\[ E Y = \frac{1 + 2 + 2 + 3}{4} \]
\[ E Y = \frac{8}{4} = 2 \]

b. \[ Var X = E X^2 - (E X)^2 = 18 - 4^2 = 10 \]
\[ sd X = \sqrt{Var X} = \sqrt{10} \]

c. \[ Var Y = \frac{48}{4} - 2^2 = 8 \]

d. \[ E(4 X - Y + 3) = (addition \ rule \ of \ E) \]
\[ 4E X - E Y + 3 = 4(4) - 3 + 3 = 17 \]

e. If X, Y are INDEPENDENT,
\[ Var(4 X - Y + 3) = \]
\[ = Var 4X + Var(-Y) + Var \]
\[ = 16 Var X + (-1)^2 Var Y \]
\[ = 16(2) + 8 \]

\[ Var(4 X - Y + 3) = 2^2 + 4^2 + 4^2 + 6^2 \]
\[ = 2^2(\frac{1}{4}) + 4^2(\frac{1}{2}) + 6^2(\frac{3}{4}) \]
\[ = 72/4 = 3\frac{6}{2} = 18 \]
7. $E X = -0.60$ and $Var X = 9$.

$T = X_1 + X_2 + ... + X_{10000}$ (independent plays)

a. $E T = E X_1 + E X_2 + ... + E X_{10000} = \frac{-0.6}{10000} = -0.00006 = E T$

if independent r.v.

b. $Var T = Var X_1 + ... + Var X_{10000} = \sum Var X_i = 10000 \times 9 = 90000$

$c. \sigma_T = \sqrt{Var T} = \sqrt{90000} = 300 \approx 0.03$

c. Approximate distribution of $T$. (CLT "central limit theorem).