1. $P(A) = 0.7$, $P(B) = 0.4$, $P(AB) = 0.2$.

   a. Venn diagram. **Hint:** $P(A B^c) = P(A) - P(AB)$

   $AB = A \cap B$

   "INTERSECTION"

   

   b. Tree diagram. **Hint:** $P(B \mid A) = \frac{P(AB)}{P(A)}$.

   

   $P(B \mid A) = \frac{0.2}{0.7}$

   total PR law $P(AB) = 0.7 \cdot 0.2 = 0.2$

   $P(AB^c) = 0.7 \cdot \frac{5}{7} = 0.5$

   $P(A^c \mid B) = 0.3 \cdot \frac{2}{3} = 0.2$

   $P(B^c \mid A) = \frac{0.1}{0.3}$

   $P(AC^c) = 0.2 \cdot \frac{1}{1-0.7} = 0.1$
1. \( P(A) = 0.7, P(B) = 0.4, P(AB) = 0.2 \).

   c. From the Venn Diagram, find \( P(B) \) and \( P(A \mid B) \).
   
   \[
P(B) = P(AB) + P(A^C B), \quad P(A \mid B) = \frac{P(AB)}{P(B)}
   \]
   
   \[
   = \frac{0.2}{0.4} + \frac{0.2}{0.4} = 0.5 \text{ and } \frac{0.2}{0.4} = 0.5
   \]

   d. From the Tree Diagram determine \( P(A \mid B) \) (Bayes).
   
   \[
P(A \mid B) = \frac{P(AB)}{P(B)}
   \]
   
   \[
   P(B) \text{ NEE ALL PIECES PASSING THRU } B
   \]
   
   \[
P(AB) = 0.2
   \]
   
   \[
P(A^C B) = 0.5
   \]
   
   \[
P(A^C B^C) = 0.1
   \]
2. \( P(\text{OIL}) = 0.1, P(+) \mid \text{OIL} = 0.7, P(+) \mid \text{no OIL} = 0.2. \)

a. Tree.

b. \( P(+) = 0.7 + 0.2 = 0.92 \)

\[ P(\text{OIL} \mid +) = \frac{P(\text{OIL} \mid +)}{P(+)} = \frac{0.7}{0.92} \]

\[ = 0.7657 \]

c. Costs: test = 20, drill = 60. Gross from oil = 400.

\[ E(\text{NET I}) = \sum x f(x) = 340(0.1) + 60(-0.9) = -60 + 400 = 340 \]

\[ E(\text{NET II}) = \sum x f(x) = 0.7 (320) + 0.3 (-20) + 0.2 (-80) + 0.8 (20) \]

\[ = 224 \]
a. What is the approximate probability of landing on Boardwalk (or any other property) in Monopoly? \( \approx \frac{1}{5} \) (See why later)

b. If the rent on that property is $200 what is the expected return to the owner from one player-circuit of the board?

\[ \text{租金 } \quad 200 \quad 0 \quad \text{期望值 } \quad E[X] = \frac{200}{0} \]

\[ \text{租金 } \quad \frac{1}{5} \quad \frac{1}{6} \quad \text{期望值 } \quad E[X] = \frac{200}{0} \]

c. If a player owns properties with rents $100, $150, $300 what is the expected return from three player-circuits of the board?

\[ 100 \left( \frac{1}{3} \right) + 150 \left( \frac{1}{6} \right) + 300 \left( \frac{1}{5} \right) \]
4. \( P(A) = 0.4, P(B) = 0.5, P(AB) = 0.20 \).

a. \( P(A \cup B) \).
\[
P(A \cup B) = P(A) + P(B) - P(AB)\text{ always.}
\]
\[
= 0.4 + 0.5 - 0.2 = 0.7
\]

b. From definition \( P(B \mid A) \).
\[
P(B \mid A) = \frac{P(AB)}{P(A)}.
\]
\[
\frac{0.2}{0.4} = \frac{1}{2}
\]

c. Are A, B independent of each other? Show reasoning! Does \( P(AB) = P(A)P(B) \)?

\[
\begin{align*}
\text{Or check: } & \quad P(B \mid A) = P(B) \ ? \\
& \quad 0.5 \neq \frac{1}{2} \\
& \quad \text{Yes, given} \\
& \quad A, B \text{ are indep.}
\end{align*}
\]
5. \( P(A) = 0.4, P(B) = 0.3, P(B \mid A) = 0.6. \)

a. Give \( P(AB) \).
   \[
P(AB) = P(A) \cdot P(B \mid A) \text{ always if } P(A) > 0.
   
   \[
P(AB) = 0.4 \cdot 0.6 = 0.24
   
   b. Are A, B independent? Is \( P(B) = P(B \mid A) \)?

   \[
   0.3 \neq 0.24 \quad \text{No - A, B are dependent}
   
   c. Fill out a complete Venn Diagram.
6. \( X = \text{draw from } \{2, 4, 4, 6\}. \ Y \text{ draw from } \{2, 2, 2, 6\}. \)

a. \( E \, X = \frac{2}{4} + \frac{4}{4} + \frac{4}{4} + \frac{6}{4} = \frac{16}{4} = 4 \)

b. \( Var \, X = E \, (X^2) - (E \, X)^2 \]
\[
E \, (X^2) = \frac{2}{4} \cdot 2^2 + \frac{4}{4} \cdot 4^2 + \frac{4}{4} \cdot 4^2 + \frac{6}{4} \cdot 6^2 = 1 + 8 + 9 = 18
\]
\[
Var \, X = 18 - 4^2 = 2
\]
\[
sd \, X = \sqrt{Var \, X} = \sqrt{2}
\]

c. \( E \, Y = \frac{12}{4} = 3 \)
\[Var \, Y = 12 - 9 = 3\]

d. \( E(4 \, X - Y + 3) = \text{(addition rule of } E)\)
\[4 \, E \, X - E \, Y + 3 = 4 \cdot (4) - 3 + 3 = 16\]

REGARDLESS OF DEPENDENCE

e. If \( X, Y \) are INDEPENDENT,
\[
Var(4 \, X - Y + 3) = Var(4 \, X) + Var(-Y) = 16 \cdot \left(2^2 + (-1)^2 \cdot 3\right)
\]
\[= 32 + 3\]
7. \( E \ X = -$0.60 \) and \( Var \ X = $9. \)
\( T = X1 + X2 + \ldots + X10000 \) (independent plays)

a. \( E \ T = E \ X1 + E \ X2 + \ldots + E \ X10000 = 10000 \times -$0.60 = -$6000 \)

if independent r.v.

b. \( Var \ T = Var \ X1 + \ldots + Var \ X10000 = 10000 \times 9 \)

\( \sigma_T = \sqrt{Var \ T} = \sqrt{10000 \times 9} = \sqrt{10000} \times \sqrt{9} = 300 \)

c. Approximate distribution of \( T \). (CLT "central limit theorem").

\[
\begin{align*}
\text{CLT} & \\
\sim & \\
\$ -6000 & \\
\end{align*}
\]