STT 200 4-13-09

We can at this point recall of lots of tests:

**Example A:** \( H_0: \mu = \mu_0 \) \( H_1: \mu > \mu_0 \)

Test Statistic:

\[
\frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \sim Z
\]

Reject \( H_0 \) if \( Z \) is large

**Example B:** \( H_0: \mu_1 = \mu_2 \) \( H_1: \mu_1 \neq \mu_2 \)

Test Statistic:

\[
\frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim Z
\]

Reject \( H_0 \) if test statistic is far from zero

But we'll not go down this path

Instead CH 26

**CH 19+20 focused on tests**

(a) \( H_0: p = p_0 \) \( H_1: p > p_0 \)

(b) \( H_0: p = p_0 \) \( H_1: p < p_0 \)

(c) \( H_0: p = p_0 \) \( H_1: p \neq p_0 \)

Test Statistic same in all cases:

\[
\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \sim Z
\]

(a) Reject \( H_0 \) if test statistic too large

(b) Reject \( H_0 \) if test statistic too small

(c) Test statistic too far from zero.
THREE SUCH CHI SQ TESTS IN YOUR READINGS.

1) TYPED BY

\[ H_0: \text{H}_0 \text{ NOT TRUE} \]

\[ H_0: \text{8 27 12 8 TOT} \]

MODEL(?) \[ SS 55 \frac{55}{4} \frac{55}{4} \frac{55}{4} \frac{55}{4} \]

chi-sq statistic

\[ \sum \frac{(\text{obs} - \text{exp})^2}{\text{exp}} \]

\[ \text{chi-sq stat for this data:} \]

\[ \chi^2 \text{TABLE:} \text{df} = 4-1 = 3 \]

\[ \chi^2 \text{TAKE:} \]

\[ (8-\frac{55}{4})^2 \]

\[ (27-\frac{55}{4})^2 \]

\[ (12-\frac{55}{4})^2 \]

\[ (8-\frac{55}{4})^2 \]

\[ \frac{55}{4} \]

\[ \frac{55}{4} \]

\[ \frac{55}{4} \]

DF(DFE GD FREEDOM) = 17.8

\[ (\chi^2)\text{SQ STAT} = \# \text{CATEGORIES} - 1 \]
\[ X^2 \text{ Stat} = 17.8 \]
\[ \text{P Value = Right Tail Pr.} \]
\[ \text{df} \quad 0.1 \quad 0.05 \quad 0.025 \quad 0.01 \quad 0.005 \]
\[ \frac{4-1=3}{6.2 \quad 7.8 \quad 9.3 \quad 11.3 \quad 12.8} \]
\[ \text{P} < \cdot .001 \quad \text{OF TABLE} \]

**Conclusion:** Either Model is Wrong OR AN EVENT OF P < .001 - OCCURRED.

Another example of some type.

<table>
<thead>
<tr>
<th>Model Customers Make Menu Choices</th>
<th>Expected Counts</th>
<th>Sample of 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>30</td>
<td>100 sample</td>
</tr>
<tr>
<td>Item 2</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Item 3</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Today you run this model sampling of 100 customers:

36 32 32 100 samples

85 counts
\[ \chi^2 \text{ stat} = \sum \frac{(\text{obs} - \text{exp})^2}{\text{exp}} \]

\[ = \frac{(36-30)^2}{30} + \frac{(32-40)^2}{40} + \frac{(32-30)^2}{30} \]

\[ = \frac{36}{30} + \frac{64}{40} + \frac{4}{30} = 1.2 + 1.6 + 5 \times 0.04 \approx 2.8 \]

\[ \text{DF} = 3 - 1 = 2 \]

\[ \text{P-value} \approx 0.10 \]

\[ 3 - 1 = 2 \] 0.04 < 5.9 ...

\[ \text{Not much evidence against } H_0 \]
Second type of $\chi^2$ test: 
\[
\chi^2 = \frac{(10 - 19.21)^2}{19.21} + \ldots + \frac{(12 - 23.21)^2}{23.21}
\]

Homogeneity (Independence)

Say: 
1. Sweatsuit 2. Not

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweatsuit</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Not</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

10: Same rates of choice for men as for women.

If indep: 
\[p(m) = \frac{\text{counts}}{\text{tot}}\]

Expected counts if sex indep of sweatshirt:

\[
\begin{array}{c|c|c|c}
   & M & F & \text{Tot} \\
--- & --- & --- & --- \\
Sweatsuit & 19.21 & 23.21 & 21 \\
Not & 23.21 & 23.21 & 21 \\
--- & 21 & 21 & 42 \\
\end{array}
\]

\[\text{DF} = (R-1)(C-1) = (2-1)(2-1) = 1\]
Another Example

Test (INCEP)

<table>
<thead>
<tr>
<th>OBS</th>
<th>62</th>
<th>707</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>39</td>
<td></td>
</tr>
</tbody>
</table>

Calc $X^2 = \sum \frac{(OBS - EXP)^2}{EXP}$

$= \frac{(14 - \frac{39.62}{112})^2}{39.62} + \ldots$

$= \frac{39.62}{112}$