STT 200  Lecture Outline 1 - 26 - 09

Prep for Exam 1 (2-3-09 in recitation).

Exercises 1, 2, 3, 4 are due in recitation 1-27-09. I will go over some or all of them in the lecture 1-26-09. The rest will be gone over in subsequent lectures.

1. A random sample of 100 (currently) enrolled MSU undergraduates is selected with the objective of estimating the fraction of MSU undergraduates who own iPods. Of these 100 there are 23 who own iPods.

1a. What is the population?

1b. What is the point estimate of p?

1c. What is the estimated standard error of \( \hat{p} \)?

1d. What is the estimated margin of error of \( \hat{p} \)?

1e. What is the 95% CI for \( p \)?

1f. What is the approximate probability that a 95% CI for \( p \) will cover \( p \)?

1g. Is it known whether or not the particular CI (1e) has covered \( p \)?

1h. If you have access to a list of all currently enrolled MSU undergraduates and the list is serially numbered from 1 to 38687 use the following random digits (in the most obvious way, scanning blocks of 5 row-wise) to select the first four of the sample of 100. Explain.

37299 07196 98642 37299 23185 56282 69929 14125 38872
71622 35940 81807 59225 18192 08710 80777 84395 69563
1i. Has the FPC (finite population correction) \[\sqrt{\frac{N-n}{N-1}}\] had much impact on the 95% CI for \(p\)?

2. A random sample of 50 (currently) enrolled MSU undergraduates is selected with the objective of estimating the average tuition of all MSU undergraduates. For these 50 students
   - sample mean = 8162.86
   - sample standard deviation = 1105.27

1a. What is the population?

1b. What is the point estimate of \(\mu\)?

1c. What is the estimated standard error of \(\bar{x}\)?

1d. What is the estimated margin of error of \(\bar{x}\)?

1e. What is the 95% CI for \(\mu\)?

1f. What is the approximate probability that a 95% CI for \(\mu\) will cover \(\mu\)?

1g. Is it known whether or not the particular CI (1e) has covered \(\mu\)?

1h. What is the 68% CI for \(\mu\)?

1i. If 100 students each undertakes a random sampling of 50 MSU undergraduates (students using entirely different parts of the table of random digits) and each of the 100 students makes a 68% CI for \(\mu\) based on their sample, around how many of the 100 students' CI will actually cover the true population mean tuition \(\mu\)?
1j. If the tuition of MSU undergraduates is distributed "approximately normal" and INSTEAD the sample mean and sample standard deviation data were from a random sample of only 5 students, what would the 95% CI be?

1k. Refer to 1j. What is the EXACT chance the 95% CI covers μ if the population distribution is perfectly normal and all calculations, including the table entry, are done with infinite precision?

1l. What are the name, pseudonym, and beer brand associated with the person credited with the method used in 1j?

1m. (unrelated to above) Sketch a normal (bell) curve having μ = 100 and standard deviation σ = 15. Clearly indicate the mean and standard deviation as recognizable elements of your sketch. In particular, identify the role of the steepest part of the normal curve. To help, I've sketched it. Identify also the 95% interval range on the horizontal axis.
3-4. A random sample of 50 male undergraduates is selected and, independently of this, a random sample of 80 female MSU undergraduates is selected.

3. Our samples find
16 males have iPods 
14 females have iPods

3a. What is the point estimate of the difference $p_{\text{male}} - p_{\text{female}}$ between the rates of iPod ownership in the populations of male and female MSU undergraduates?

3b. What is the estimated standard error of $\hat{p}_{\text{male}} - \hat{p}_{\text{female}}$?
3c. What is the estimated margin of error of \( \hat{p}_{\text{male}} - \hat{p}_{\text{female}} \)?

3d. The sample sizes of 16 and 14 are rather small, so the approximate coverage probability of a 95% CI may not be close to the intended 0.95. For the purpose of illustration, what is the 95% CI for \( p_{\text{male}} - p_{\text{female}} \)?

4. Our samples of 50 and 80 find
   sample mean tuition (males) = 8322.80
   sample mean tuition (females) = 7993.06

   sample standard deviation (males) = 1322.97
   sample standard deviation (females) = 1164.79

4a. What is the point estimate of the difference \( \mu_{\text{male}} - \mu_{\text{female}} \) between the average tuition for the men and women MSU undergraduates?

4b. What is the estimated standard error for \( \bar{x}_{\text{male}} - \bar{x}_{\text{female}} \)?

4c. What is the estimated margin of error for \( \bar{x}_{\text{male}} - \bar{x}_{\text{female}} \)?

4d. What is the 95% CI for \( \mu_{\text{male}} - \mu_{\text{female}} \)? Admittedly, our sample sizes are too small to justify the method.
5. Use the z-table to determine the value of z for which the area left of z is 0.9990. It is the z-score surpassing 99.9% of all z-scores (area left of this z is 0.9990). When you have found this z you can use it to find the 99.9th percentile of IQ. Suppose IQ scores are normal distributed with mean 100 and standard deviation 15. The relation you need is:

\[ \text{IQ} = \mu + z \sigma = 100 + z \times 15 \]

Find the z-score from the table and use it as above to discover the IQ at 99.9th percentile of IQ.

6. Two experimenters independently sample data to estimate the same thing, employing 1.96 in calculating their estimated margins of error (based on large sample methods)

- estimated margin of error obtained by experimenter A = 1.356
- estimated margin of error obtained by experimenter B = 1.288

What estimated margin of error would you quote for the difference between their respective point estimates (not given)? What famous Greek is associated with the rule being used?

7. We've randomly sampled several tiny granules whose weights x have been carefully measured. The population distribution of the weights of granules is thought to be normal. The sample measurements are: 1.2278 1.2274 1.2269.

7a. Calculate the sample mean.

7b. Calculate the sample standard deviation.

7c. What is the effective size of N? What affect does it have on the value of \( \sqrt{\frac{N-n}{N-1}} \)?

7d. Determine the 99% (not 95%) CI for the population mean granule weight \( \mu \).
8. (We will cover this new idea by Friday) It is desired to estimate the size of a crowd from a photograph taken from above. The area is divided up into 1056 equal patches. The total number of people is therefore 1056 times the average number of people per patch. A random sample of 100 patches is selected from which are found by careful examination of the photo the numbers \( x_i = \text{number of people in patch \( i \)} \), \( 1 \leq i \leq 100 \). From these scores are calculated

\[
\bar{X} = \frac{\sum_{i=1}^{100} x_i}{100} = \text{sample mean number of people per patch} = 62.55
\]

sample standard deviation \( s = 23.49 \)

8a. What do you think is a reasonable estimate of the total crowd size 1056 \( \mu \)?

8b. What do you think is a reasonable estimate of the margin of error of your estimate 8a?

8c. Give the 95% CI for the total crowd size.