Day Ch 15. Recall Ch 14

Classical Pr model:

VENN

UNIVERSE (SAMPLE SPACE)

All 10 eq pr.

P(A U B) = P(A) + P(B) - P(A \cap B)

= 2/3 4 2 3 3 4

P(A U \emptyset) = P(A) + P(\emptyset) = \frac{2}{7} + \frac{3}{7}
**General Addition Rule**

\[ P(A \cup B) = P(A) + P(B) - P(A \text{ and } B) \]

**Attack**

\[ P(A \text{ and } B) \]

**Classical**

\[
\frac{\#A \cap B}{\#\text{TOTAL}} = \frac{\#A}{\#\text{TOTAL}} \times \frac{\#A \cap B}{\#A}
\]

\[ P(A \text{ and } B) = P(A) \times \frac{\#A \cap B}{\#A} \]

\[ \frac{\#A \cap B}{\#A} = \frac{\#A \cap B}{\#\text{TOTAL}} \times \frac{\#A}{\#\text{TOTAL}} = \frac{P(A \cap B)}{P(A)} \]

So indeed **conditional probability** for B given that A was occurred.

\[ \text{CONDITIONING EVENT} \]
\[ P(B | A) \text{ def } \frac{P(ANB)}{P(A)} \text{ if this is just } P(B) \]

If events are independent, it would then be equivalent to
\[ P(ANB) = P(A)P(B) \]

Illustration of rules: add, mult, total prob

Balls:
\[ 6 R \ 3 G \ 4 Y \]

\[ P(R1) = \frac{6}{13} \]
\[
P(R_2 \mid R_1) = \frac{5}{12}
\]

\[
P(R_1 \cap R_2) = P(R_1) \cdot P(R_2 \mid R_1)
\]

\[
= \frac{6}{13} \cdot \frac{5}{12}
\]

\[
P(R_1 \cap R_2) = \frac{6 \cdot 5}{13 \cdot 12} = \frac{6}{13}
\]

\[
\text{And these } P(R_2) = \frac{6}{13} \cdot \frac{5}{12} + \frac{7}{13} \cdot \frac{6}{12} = \frac{72}{13 \cdot 12} = \frac{6}{13}
\]

\[
\text{Same as } P(R_1)
\]
redo above but draw with repl.

Intuition: \( P(R1) = \frac{6}{13}, \ P(R2) = \frac{6}{13} \).

\[
P(R1 \cap R2) = P(R1) \cdot P(R2 \mid R1) = \frac{6}{13} \cdot \frac{6}{13} \]

\[
P(R1^c \cap R2) = P(R1^c) \cdot P(R2 \mid R1^c) = \frac{7}{13} \cdot \frac{6}{13} \]

Same as \( P(R1) \)

\( \ P(R2) = \text{TOTAL} = \frac{6}{13} \)

INDEPENDENCE OF A, B

DEF: \( P(A \cap B) = P(A) \cdot P(B) \)

EXCEPT FOR CASE \( P(A) = 0 \) THIS SAYS \( P(B \mid A) = P(B) \).

CROSS OUT IF NECESSARY.
In tables, independence is seen as proportionality.

\[
\begin{align*}
\text{M} & \quad 10 & \quad 20 & \quad 30 \\
\text{F} & \quad 10 & \quad 15 & \quad \text{P(\text{AA} | \text{M})} = \frac{\text{P(\text{AA}, \text{M})}}{\text{P(M)}} = \frac{10}{60} \\
\text{P(\text{AA} | \text{F})} = \frac{5}{30} & \quad \text{same proportionality is the look of independence.}
\end{align*}
\]

**Tree Diagram**

**Pos for OIL**

\[
\begin{align*}
\text{P(\text{+ | OIL})} & \approx 0.9 \quad \text{nicely large} \\
\text{P(\text{+ | \text{NOIL})} & \approx 0.3 \quad \text{nicely small} \\
\text{P(\text{+ | OIL})} & = 0.9 \\
\text{P(\text{+ | \text{NOIL})} & = 0.29 \\
\text{P(\text{+})} & = 0.18 \\
\end{align*}
\]
\[ P(\text{oil}) \]

\[ P(\text{oil}) = 0.2 \]

\[ P(\text{oil}) = 0.2 \]

\[ P(+|\text{oil}) = 0.3 \]

\[ P(\text{oil}^-) = 0.7 \]

\[ P(-|\text{oil}^-) = 0.8 \]

\[ P(\text{oil}^-) = 0.8 \cdot 0.7 = 0.56 \]

\[ \text{Bayes} P(\text{oil}^-|+) \]

\[ \text{DEF and} \]

\[ P(\text{oil}^-|+) = \frac{0.18}{0.18 + 0.24} \]

\[ \text{Compare this to initial} \ P(\text{oil}) = 0.2 \]
\[ P(C|\text{oil} \_1 \_H) = \frac{P(C|\text{oil} - \_H \_F)}{P(C - \_H)} = \frac{0.02}{0.02 + 0.56} \]

#2.

GIVEN: 
\[ P(A) = 0.8, \quad P(B) = 0.7, \quad P(A \cap B) = 0.6 \]

\[ P(A) = 0.8 \]

\[ P(B) = 0.7 \]

\[ P(A \cap B) = P(A)P(B) \]

\[ \text{NOT} P = 0.5 \]

\[ P(C) = 0.6 \]

\[ P(\text{oil} \_1 \_H) = 0.02 \]

\[ P(\text{oil} \_1 \_F) = 0.56 \]

\[ P(\text{oil} - \_H) = 0.98 \]

\[ P(\text{oil} - \_F) = 0.44 \]

\[ P(C|\text{oil} \_1 \_H) = \frac{0.02}{0.02 + 0.56} \]

\[ P(C|\text{oil} - \_H) = \frac{0.98}{0.98 + 0.44} \]

\[ P(C|\text{oil} - \_F) = \frac{0.56}{0.98 + 0.56} \]