LEAD OF W/ OVERVIEW (*#16)

**DEF OF RANDOM VARIABLE.**
A numerical function on the outcomes of a probability experiment.

**JACK + JILL $1, $1, $5**

**RANDOM VARIABLE**
J = "AMOUNT DRAWN BY JACK"

Outcomes: $1, $1, $5

Value of J: ①, ④, ⑤

**EXPECTED VALUE**

\[ E(X) = \sum_{i=1}^{n} w_i / \# \text{STUDENTS} \]

**SAME**
\[ \sum \frac{v_i}{n} \cdot p(v) \]

**DISTINCT**
\[ \sum \uparrow \text{DISTINCT} \cdot p(v) \]

**VARIANCE**
\[ \sum (v - E(X))^2 \cdot p(v) \]

**DEVIATIONS FROM**
\[ \sum (v - \text{AVG})^2 \]
Find \( P(J = 1) = \frac{\text{# Favorable}}{\text{# Total}} = \frac{1}{3} \) \( P(J = 5) = \frac{1}{3} \)

\[
\begin{pmatrix}
\text{Values} & 1 & 5 \\
\text{p}(j) & \frac{2}{3} & \frac{1}{3}
\end{pmatrix}
\]

Distribution of random variable.

Notion of "probability weighted average value."

\[
EJ = \sum \text{p}(j) = 1 \left( \frac{2}{3} \right) + 5 \left( \frac{1}{3} \right) = \frac{7}{3}
\]

Possible value of \( J \).

Say \( EJ = \frac{7}{3} \)

Say "expected value of \( J \) is \( \frac{7}{3} \)."

But Jack either gets \( \$1 \) or \( \$5 \).

Relevance of \( EJ \) is that \( \frac{J_1 + J_2 + \ldots + J_{10000}}{10000} \approx \frac{7}{3} \).
IMPORTANT: IS A SENSE TO BE DESCRIBED,

Random sum: \( \sum V \uparrow + \cdots + \frac{V}{10000} \)

Return of venture 1: \( \uparrow \) venture 10000

Track closely with: \( E[V] \uparrow \frac{V}{10000} \)

Some conditions required

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**#4. ON RECIPIENT 1-26-10**

Lottery values: \( x \) 20 -5 0

4a. Calc. \( E[X] = \sum x \cdot p(x) = 20 (0.2) -5 (0.4) + 0 (0.4) \)

Value: \( 4 - 2 + 0 = 2 \)
46. \[ \sum_{x} (x - \bar{x})^2 \cdot p(x) \quad \text{Recall } \bar{x} = 2 \]

\[ = (20 - 2)^2 \cdot 0.2 + (-5 - 2)^2 \cdot 0.4 = 86 \]

\[ \sum_{x} x^2 \cdot p(x) = \begin{align*}
20^2 \cdot 0.2 + (-5)^2 \cdot 0.4 + 0^2 \cdot 0.4 &= 90
\end{align*} \]

**Terminology:** Variance of \( X \) \( \text{def} \sum_{x} (x - \bar{x})^2 \)

**Important Property**

\[ \text{Variance of } X = E(X^2) - (E(X))^2 \]

\( E(X) = \bar{x} \quad \text{Recall so } (E(X))^2 = 4 \)

\[ E(X^2) = \sum_{x} x^2 \cdot p(x) = \begin{align*}
20^2 \cdot 0.2 + (-5)^2 \cdot 0.4 + 0^2 \cdot 0.4 &= 90
\end{align*} \]
So truly variance of $X = 90 - 4 = 86$

By the alternate method "AVE of SQUARES" - SQUARE OF PR W/O AVE.

$90$ is AVE of SQUARES $E(X^2) = \sum x^2 p(x)$

$4$ is $(\bar{X})^2 = (\sum x p(x))^2$

$\textbf{4C. STANDARD DEVIATION} = \sqrt{\text{Variance}}$

$= \sqrt{86} = 9.27362$

$\textbf{#5. LOTTERY HAS RANDOM RETURN} X, \text{ with } E[X] = -$0.10, STANDARD DEVIATION $X = $0.80
Will play 100 times independently

5a. Claim: \( E(\text{total of 100 plays}) = E(X_1 + X_2 + \ldots + X_{100}) \)
\[ = E(X_1) + \ldots + E(X_{100}) = 100(-\$0.10) = -\$10.00 \]

5b. Variance \( (X_1 + \ldots + X_{100}) = \text{Var}(X_1) + \ldots + \text{Var}(X_{100}) \)
\[ = 100(\$0.80)^2 = 64 \]
So standard deviation \( = \sqrt{64} = 8 \)

5c. Sketch the approximate appearance of the total for 100 independent plays.

![](image.png)

Possible value of \( X_1 + \ldots + X_{100} \)
- $10.00 - 8
- $18.00

- $2.00
- $10.00 + 8

68%

95%
$\#3. \quad OIL \quad P(OIL) = 0.2 \quad P(+) | OIL = 0.9 \quad P(+) | \overline{OIL} = 0.3$

\[
P(+) = 0.18 + 0.24
\]
Bayes - update \[ P(OIL|+) = \frac{P(OIL+)}{P(+)} = \frac{.18}{.18 + .24} \]

\[ \text{Oil} \]

\[ .9 \]

\[ 3b_0 \]

\[ .2 \]

\[ .18 \text{ Oil} + -80 - 200 + 900 \]

\[ .62 \text{ Oil} - -80 - 0 + 0 \]

\[ .3 \]

\[ .24 \text{ Oil} + -80 - 200 + 0 \]

\[ .56 \text{ Oil} - -80 - 0 + 0 \]

\[ 1 \]

\[ \sum x \rho(x) = -2 \]

\[ E(\text{NET I}) = (-200 + 900)(.2) \]

\[ + (-200 + 0)(.8) = 20 \]

Policy I "JUST DRILL"

Policy II "TEST BUT ONLY DRILL IF TEST IS +"