CH 17 Probability Models

We've seen the Normal Distribution

The Bell Curve

NORMAL DISTN

GAUSSIAN DISTN

Example IQ

Mean IQ ~ 100

Std Dev IQ ~ 15

⇒ ~68% of Persons

Have IQ in range [85 - 115]

Rule of Thumb.
We will play 100 times (independent plays)

\[ \text{Rule: } E(\text{Total} + \text{Always } E(X) = X_1 + \cdots + X_{100} = 100 \times 10 = 1000} \]

Write \( \mu = 10, \sigma = 3 \)

\[ \sigma = \sqrt{\text{Var}(X)} \]

\[ \text{Single Play} = \mu = 10 \]

\[ \text{Standard Application:} \]

\[ x \sim \text{Normal (Single Play)} = 10 \Rightarrow \text{Normal (All Plays)} = 10 \]

\[ z = \frac{x - \mu}{\sigma} = \frac{10 - 10}{3} = 0 \]

\[ z = 0 > z_{1999} \]
Rule: Variance of Total = \text{Var}(X_1 + X_2 + \cdots + X_{100})

\[ = 100 \times 9 = 900 \]

\[ \text{So SD of Total} = \sqrt{\text{Var Total}} = \sqrt{900} \times 9 = 30 \]

\[ \frac{\text{Approx}}{\sim} \quad \text{Distn of Total} \sim \]

\[ \text{So around 68\% chance} \quad \text{my total is within} \quad [1000 - 30, 1000 + 30] \]

\[ \text{So to \sim 95\% chance total within} \quad [1000 - 2 \times 30, 1000 + 2 \times 30] = [940, 1060] \]

\[ \text{So normal model is useful!} \]
OTHER MODELS

1. BERNULLI TRIALS
   INDEPENDENT TRIALS
   EACH HAS 2 POSSIBLE OUTCOMES
   SUCCESS PROBABILITY \( p \)
   FAILURE PROBABILITY \( q = 1 - p \)

1a. Coin Tosses
   \( m = 100, p = 0.5 \)
   \( \text{Success} = "H" \)
1b. 1000 Patient Exams.
   \( \text{Success} = "HAS APPARENT MELANOMA" \)
   \( \text{Failure} = \text{NO} \)
   \( p \) \( \text{WHATSOEVER THE POPULATION RATE OF MELANOMA IS} \)
   \( \text{PROVIDED I SAMPLE THAT POPULATION AT RANDOM} \).
1c. Gallup Poll: e.g. Sample 1500 Voters
   \( \text{Success} = "DEM" \)
   \( \text{Failure} = "REP" \)
   \( m = 1500, S = S, F = F \)
Let \( T \) = total of \( n \) Bernoulli trials scored

\[ 1 \quad \text{"success"} \quad 0 \quad \text{not} \]

\[ X = X_1 + \cdots + X_n \quad \text{# of successes in sample of } n \]

\[ \text{Var} X = \mathbb{E}(X^2) - (\mathbb{E}X)^2 \]

\[ = \rho - \rho^2 = \rho(1-\rho) \]

So 23-27 on 2-2-10 recitation.

Toss one six-sided die

Ace \( \Delta \) = "success"  \( \rho = \frac{1}{6} \quad q = \frac{5}{6} \)
Formulas for

\[ E(X) = \mu = \frac{1}{6} \]

When you toss a die you get on avg \( \frac{1}{6} \) th of an ace.

\[ \text{Var}(X) = \sigma^2 = \frac{1}{6} \times \frac{5}{6} \]

\( t(n) \text{ (#successes)} = n \cdot \mu \)

For \( n = 100 \) tosses.

\[ t \left( \sum X_i \right) = E \left( X_1 + \ldots + X_{100} \right) = 100 \left( \frac{1}{6} \right) \approx \frac{1}{16} \text{ aces} \]

\[ \text{Var} \left( \sum X_i \right) = 100 \left( \frac{1}{6} \times \frac{5}{6} \right) = n \cdot \mu \cdot \sigma^2 \]

Book's Rule - must expect \( \geq 10 \) and \( n \cdot \text{expect} \geq 10 \)

\[ \sqrt{npq} = \sqrt{100 \left( \frac{1}{6} \times \frac{5}{6} \right)} \]

100 (\( \frac{1}{6} \)) aces in 100 tosses.
So ~ 68% chance # of aces in 100 tosses of a die is within range $[100(\frac{1}{6}) - \sqrt{100(\frac{1}{6})^2}, 100(\frac{1}{6}) + \sqrt{100(\frac{1}{6})^2}]$.

# 18-22. Fair coin tossed 100 times.

$X = \frac{1}{2}$

Total = $X_1 + \ldots + X_{100}$

Each has $E(X) = \frac{1}{2}$

$Var(X) = pq = 100(\frac{1}{2})(\frac{1}{2}) = 25$

$SD(Total) = \sqrt{Var} = \sqrt{25} = 5$

$E(Total) = np = 100(\frac{1}{2}) = 50$

$\approx$ Normal $\sim$ ~
So around 68% chance #Heads in 100 tosses is within \([50-5, 50+5] = [45, 55]\), also \(-9.5\%\) chance \([40, 60]\).

# 28-31
Sample many times \(n \to \infty\) each trial has probability \(p \to 0\).

Note \(np = \text{avg #seen}\) (usually we "know" from experience)

Variance = \(np(1-p)\) if \(p \to 0\)

So suppose we experience, on avg, around 4% hospital emergency admissions for eye injury.
\[
\Rightarrow ~ \sim \sqrt{4.7} \]

# OF ADMISSIONS FOR EYE INJURY

\[4.7\]

RULE OF THUMB
- ONLY USE THIS IF EXPECTED \( \geq 3 \).

CALLED POISSON - TOTAL # SUCCESSES IN TRIALS, \( P \)

\[n \rightarrow \infty \quad p \rightarrow 0\]

Suppose that on avg we experience 8.2 persons struck by lightning each summer season.

\[\sqrt{2}\]

\[\approx 0.16\]

\[8.2\]

\[11\]
**Poisson Choc Cookies**

**Dough** makes 144 cookies.

\[ p = \frac{1}{144} \]

\[ m = 5 \times 144 \]

\[ np = \left(5 \times 144\right) \frac{1}{144} = 5 \]

\[ \sim \text{DISTN of # of choc pieces in a cookie} \]

\[ \sim \text{N(5)} \]
SEED DISTRIBUTION

AVG 20 SEEDS PER SQ FOOT

\[ \sqrt{20} \]

20

# SEEDS PER SQ FOOT

POISSON
(NORMAL APPROX OF)