Note Title 1/27/2010

STT 200 3pm 1-27-10

Ch 16 + Ch 17 (part)

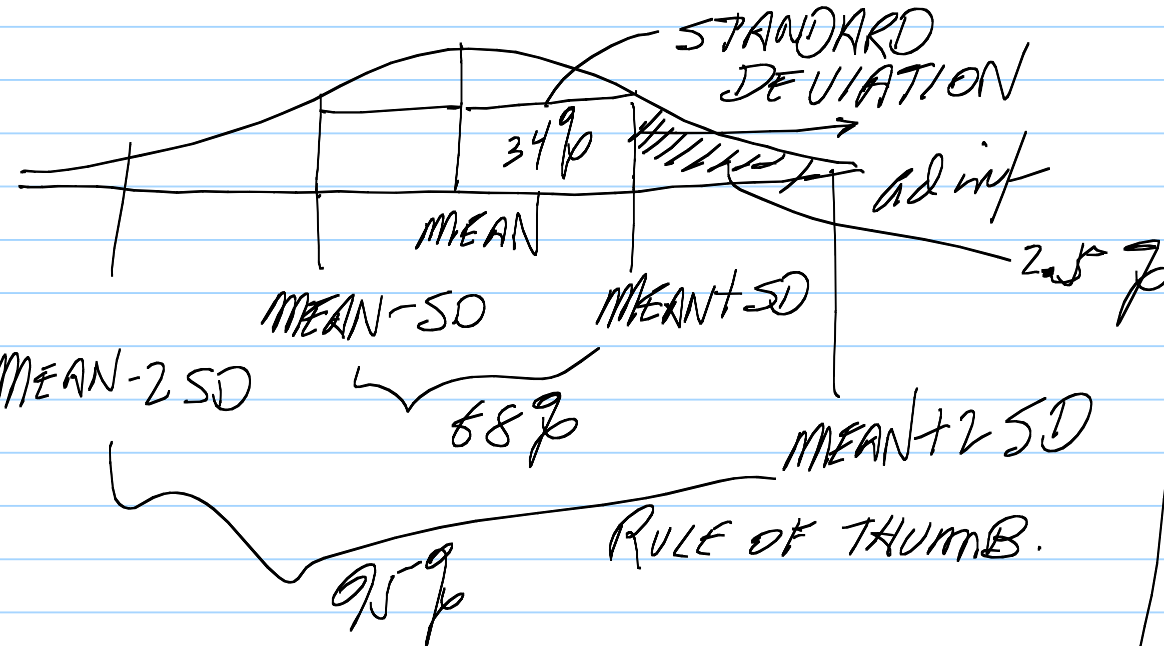
Ch 17 PROBABILITY MODELS

WE'VE SEEN THE NORMAL DISTRIBUTION

i.e. Bell Curve

NORMAL DIST^N

GAUSSIAN DIST^N



RULE OF THUMB.

EXAMPLE IQ.

MEAN IQ ~ 100

STD DEV IQ ~ 15

⇒ ~68% OF PERSONS HAVE IQ IN RANGE [100-15, 100+15] = 85 TO 115.

App' OF NORMAL MODEL (AS AN APPROXIMATION)

STANDARD SUCH APPLICATION:

HAVE LOTTERY EX (SINGLE PLAY) $= 10 = \sum_x x\,p(x)$

σ_X ~ σ ~ $SD\,X$ (") $= 3 = \sqrt{\sum_x (x - EX)^2 p(x)}$

(SIGMA)

ALSO $= \sqrt{E(X^2) - (EX)^2}$

WRITE $\mu = 10$ $\sigma = 3$

"mu" "sigma"

Spse WE WILL PLAY 100 TIMES (INDEPENDENT PLAYS)

TOTAL $= X_1 + \cdots + X_{100}$

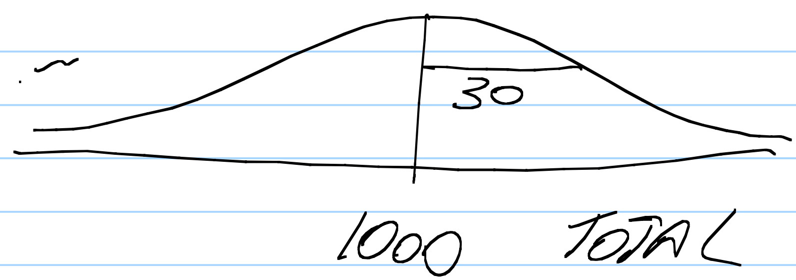
RULE E TOTAL $\overset{\text{ALWAYS}}{=} EX_1 + \cdots + EX_{100} = 100(10) = 1000$

RULE Variance of TOTAL $\overset{\text{INDEPENDENCE}}{=}$ $\text{Var } X_1 + \text{Var } X_2 + \cdot + \text{Var } X_{100}$

$= 100(9) = 900$

SO SD of TOTAL $= \sqrt{\text{Var TOTAL}} = \sqrt{100 \; 9} = 30$

AND $\underset{\text{APPROX}}{\approx}$ DIST^N OF TOTAL $\sim$



SO AROUND 68% CHANCE MY TOTAL IS WITHIN $[1000-30, 1000+30]$

SO TO ~95% CHANCE TOTAL WITHIN RANGE $[1000 - 2\;30, \; 1000 + 2\;30] = [940, 1060]$.

SO NORMAL MODEL IS USEFUL!

OTHER MODELS

(1) BERNOULLI TRIALS

n INDEPENDENT TRIALS.
EACH HAS 2 POSSIBLE OUTCOMES

SUCCESS PROBABILITY p
FAILURE PROBABILITY $q = 1 - p$

$\underline{\qquad p + q \qquad}$
1

1a. 100 COIN TOSSES $n = 100$, $p = .5$
SUCCESS = "H"

1b. 1000 PATIENT EXAMS.
SUCCESS = "HAS APPARENT MELANOMA"
FAILURE = NOT
p? WHATEVER THE POPULATION RATE OF MELANOMA IS
+ PROVIDED I SAMPLE THAT POPULATION AT RANDOM.

1c. GALLUP POLL : eg. SAMPLE 1500 "VOTERS" SUCCESS "DEM"
FAILURE "REP"
$n = 1500$, S F S F F F

LET T = TOTAL OF n BERNOULLI TRIALS SCORED

$$\begin{array}{ll} 1 & \text{"SUCCESS"} \\ 0 & \text{NOT} \end{array}$$

$$T = X_1 + \cdots + X_n$$

= # OF SUCCESSES IN SAMPLE OF n

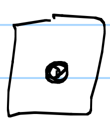
$$X \quad \begin{array}{lll} 1 & p & \text{"SUCCESS"} \\ 0 & q & \text{NOT} \end{array}$$

$$EX = 1p + 0q = p$$

$$\mathrm{Var}\, X = E(X^2) - (EX)^2$$
$$= p - p^2 = p(1-p)$$
$$= pq$$

SO 23-27 ON 2-2-10 RECITATION.

TOSS ONE SIX-SIDED DIE

ACE  = "SUCCESS" $\quad p = 1/6 \quad q = 5/6$

FORMULAS FOR BERNOULLI TRIAL

$$E X = p = \frac{1}{6}$$

WHEN YOU TOSS A DIE YOU GET ON AVG $1/6$ TH OF AN ACE.

$$\mathrm{Var}\, X = pq = \frac{1}{6}\,\frac{5}{6}.$$

$E(\#\text{SUCCESSES}) = np$

FOR $n = 100$ TOSSES. $\sim \begin{matrix} 1 & 1/6 \\ 0 & 5/6 \end{matrix}$

$$E(\text{TOTAL \# ACES}) = E(X_1 + \cdots + X_{100}) = 100\left(\frac{1}{6}\right) \approx 16 \text{ ACES}$$

$$\mathrm{Var}(\text{TOTAL \# ACES}) = 100\left(\frac{1}{6}\,\frac{5}{6}\right) = npq$$

BOOK'S RULE — MUST EXPECT ≥ 10 AND $n -$ EXPECT ≥ 10

OR $\Longrightarrow$ $\sim$

$\sqrt{npq} = \sqrt{100\left(\frac{1}{6}\right)\frac{5}{6}}$

$100(1/6)$ # ACES IN 100 TOSSES.

SO ~68% CHANCE # OF ACES IN 100 TOSSES OF A DIE IS WITHIN RANGE $\left[100\left(\frac{1}{6}\right) - \sqrt{100\left(\frac{1}{6}\right)\frac{5}{6}},\ 100\left(\frac{1}{6}\right) + \sqrt{100\left(\frac{1}{6}\right)\frac{5}{6}}\right]$.

#18-22. FAIR COIN TOSSE 100 TIMES.

$$X = \begin{cases} 1 & H & \frac{1}{2} \\ 0 & T & \frac{1}{2} \end{cases}$$

$TOTAL = X_1 + \cdots + X_{100}$

EACH HAS

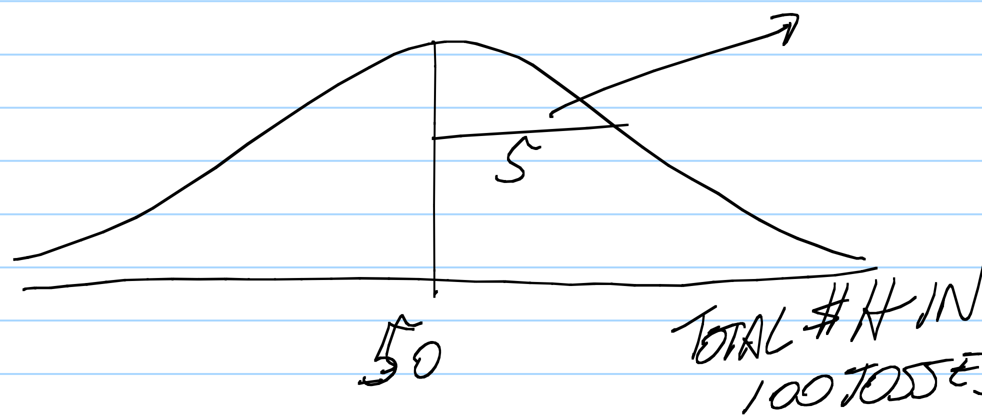
$EX = p$

$Var X = pq \implies \sim$

$E(TOTAL) = np = 100\left(\frac{1}{2}\right) = 50$

$Var(TOTAL) = npq = 100\left(\frac{1}{2}\right)\frac{1}{2} = 25$

$SD(TOTAL) = \sqrt{Var} = \sqrt{25} = 5$



SO AROUND 68% CHANCE # HEADS IN
100 TOSSES IS WITHIN $[50-5, 50+5] = [45, 55]$.
ALSO ~95% CHANCE $[40, 60]$.

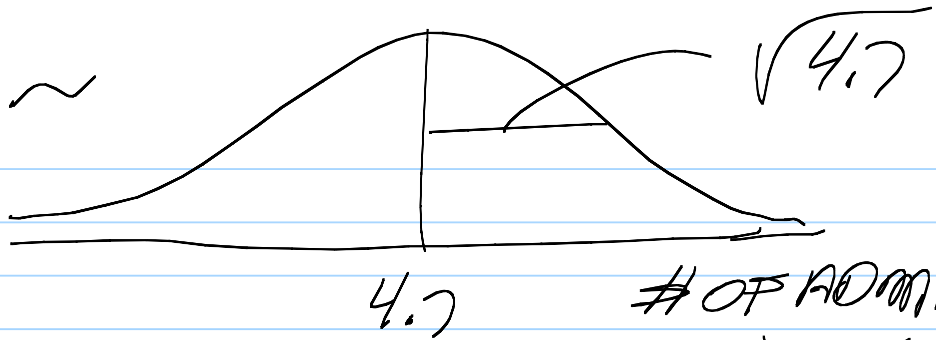
#28-31 SAMPLE MANY TIMES $n \sim \infty$
EACH TRIAL HAS PROBABILITY $p \sim 0$.

NOTE np = AVG # SEEN (USUALLY WE "KNOW" FROM EXPERIENCE)

Variance $= np\underbrace{q}_{\sim 1 \text{ IF } p \sim 0}$

SO SUPPOSE WE EXPERIENCE, ON AVG, AROUND 4.7
HOSPITAL EMERGENCY ADMISSIONS FOR EYE INJURY.

$\Rightarrow$ $\sim$

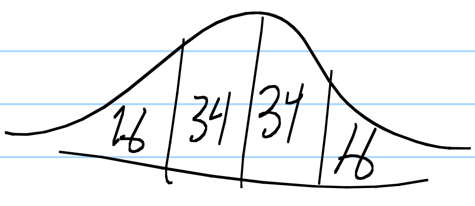


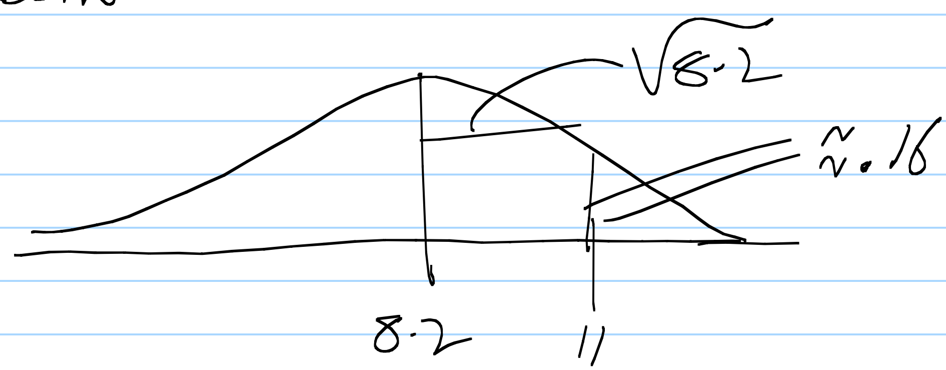
RULE OF THUMB
— ONLY USE THIS IF EXPECTED ≥ 3.

OF ADMISSIONS FOR EYE INJURY

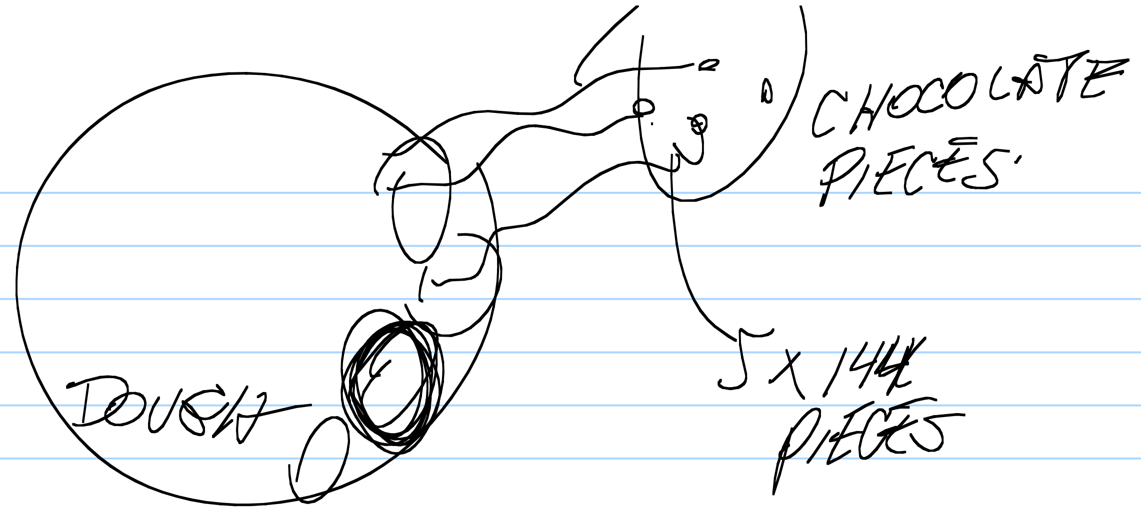
CALLED POISSON — TOTAL # SUCCESSES n TRIALS, p
$n \sim \infty$ $p \sim 0$

SUPPOSE THAT ON AVG WE EXPERIENCE 8.2 PERSONS STRUCK BY LIGHTNING EACH SUMMER SEASON.





POISSON CHOC COOKIES



DOUGH MAKES 144 COOKIES.

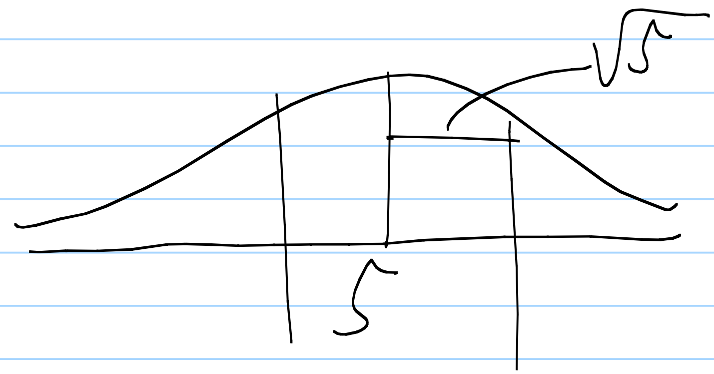
$$p = \frac{1}{144}$$

$$n = 5 \times 144$$

$$np = (5 \times 144)\frac{1}{144} = 5$$

≈ DIST^N OF # OF CHOC PIECES IN A COOKIE

≈



SEED DISTRIBUTION

AVG 20 SEEDS PER SQ FOOT

POISSON

(NORMAL APPROX OF)