STT 200, 5:30pm 2-1-10a

Note Title 2/1/2010

TODAY: EARLY PARTS OF RECITATION 2-2-10 AS PART OF A REVIEW OF OTHER (NEW) MATERIAL PER EXAM 1 NEED.

1-12. PROBABILITY DISTRIBUTION = LIST OF $(x, p(x))$ (DISCRETE VALUES)

$p(x) = P(X = x)$

($p(x)$: lowercase; X: uppercase, random variable)

x	$p(x)$	$xp(x)$	$x^2p(x)$
0	.2	0	$0^2(.2) = 0$
1	.3	.3	$1^2(.3) = .3$
-2	.5	-1	$(-2)^2(.5) = 2$
TOTAL	1	$EX = -0.7$	$E(X^2) = 2.3$

$(x - EX)^2 p(x)$

$(0 - (-0.7))^2 \cdot .2 = .49(.2)$

$(1 - (-0.7))^2 \cdot .3 = 2.89(.3)$

$(-2 - (-0.7))^2 \cdot .5 = 1.69(.5)$

$\text{Var}\,X \underset{\text{def}}{=} E(X-EX)^2 = 1.81 \underset{\text{SHORT FORM}}{=} \text{Var}\,X = E(X^2) - (EX)^2 = 2.3 - (-0.7)^2 = 1.81$ ✓

eg $\text{Var}(aX) = a^2 \text{Var}\,X$

$E(aX)^2 = Ea^2X^2 = a^2EX^2$

$(E(aX))^2 = (aEX)^2 = a^2(EX)^2$

DIFF $\text{Var}(aX) = a^2 \text{Var}\,X$ ✓

RULES: $E(aX+bY+c) \underset{\text{ALWAYS}}{=} aEX + bEY + c$

$\text{Var}(aX+c) = a^2 \text{Var}\,X$

$\sigma_X = SDX = \sqrt{\text{Var}\,X}$

$\sigma_{aX} = \sqrt{a^2\sigma_X^2} = |a|\sigma_X$

1. $EX = -0.7$
2. $\text{Var}\,X = 1.81$
3. $EX^2 - (EX)^2 = 1.81$
4. $E(3X+7) = 3EX + 7 = 3(-0.7) + 7$

5. $E(3X - X + 7) = E3X - EX + 7 = 3EX - EX + 7 = 2EX + 7$

$E(2X+7) = 2EX + 7$

6. $\mathrm{Var}(3X+7) = 9\,\mathrm{Var}X = 9(1.81)$

7. $\mathrm{Var}(3X - X + 7) \overset{\text{NOT!}}{=} \mathrm{Var}3X + \mathrm{Var}(-X) + 0$ ↑ NOT INDEP.

8. r.v. $EY = 6$ so $E(X - 2Y + 4) \underset{\text{ALWAYS}}{=} EX - 2EY + 4$

$= -0.7 - 2(6) + 4$

$\mathrm{Var}(-Y) = (-1)^2 \mathrm{Var}Y$

9. Y end of X, $\mathrm{Var}Y = 2$ $\mathrm{Var}(5X - Y + 7) \underset{\text{INDEP}}{=} 25\,\mathrm{Var}X + 2$

10. $SD\,X \underset{DEF}{=} \sqrt{Var\,X} = \sqrt{1.81} = \sigma_X$

11. $SD\,of\,(3X - 2X + 4) = \sqrt{Var(X+4)} = \sqrt{1.81}$ (DEPENDENT.)

12. Recall $SD(5X - Y + 4) = \sqrt{Var(5X - Y)}$

$\overset{INDEP}{=} \sqrt{25\,Var\,X \oplus Var\,Y} = \sqrt{25(1.81) \oplus 2}$

PHRASE TO LOOK AT "PARADOX OF FALSE POSITIVE" (GENIC? SMITH)

	DISEASE	NOT	
+	1	20	21
−	0	979	979
	1	999	1000

$P(+|D) = \frac{1}{1} = 1$ $\quad P(+|D^c) = \frac{20}{999} \sim .002$

$P(D|+) = \frac{1}{21} \sim .05$

PER EXAM: POISSON BINOMIAL AGGREGATION OF INDEP LOTTERIES

IF POISSON RANDOM VARIABLE X = # OF RARE EVENTS

$X = 0, 1, 2, \ldots$

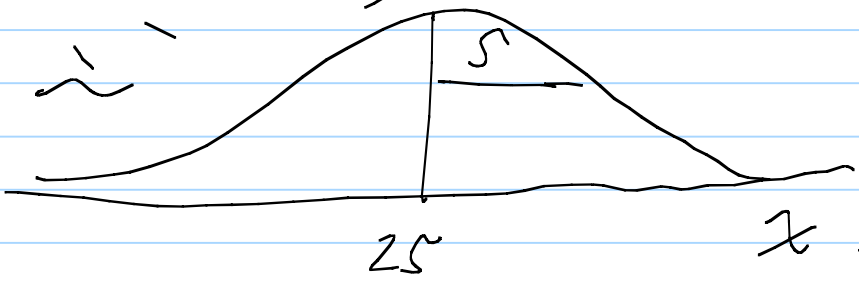
SUPPOSE THAT WE AVERAGE 25 "ACCIDENTS."

$EX \sim 25$ POISSON $\mathrm{Var}\, X = EX$ $SD\, X = \sigma_X = \sqrt{25} = 5$

FOR POISSON $SD\, X = \sqrt{EX}$

ALSO: IF $EX \geq 3$, WE SUGGEST NORMAL APPROX

$\sim$



68% INTERVAL $[25-5, 25+5]$

95% : $[25-10, 25+10]$

$X = 0, 1, 2 \ldots$

2. BINOMIAL REPEATED TRIALS. SUPPOSE 70% OF VOTERS VOTE OPPOSITE THEIR INTEREST.

SAMPLE (INDEP) 400 VOTERS.

X = # OF SAMPLE VOTERS VOTING OPPOSITE THEIR INTEREST.

$0, 1, \ldots, 400.$

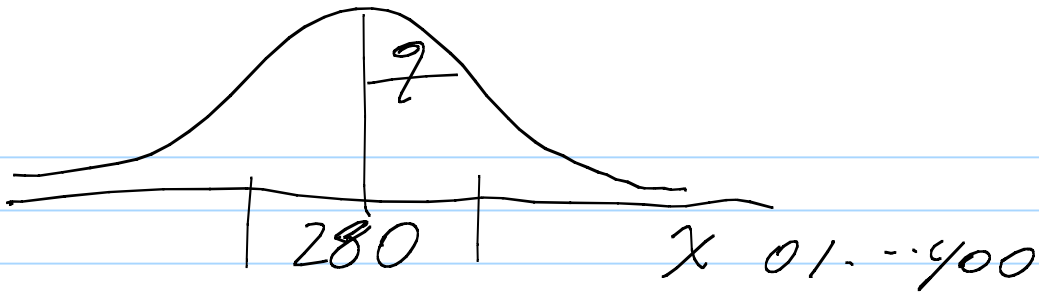
$n = 400 \quad p = .7 \quad q = .3$

$EX = 280 = np = 400(.7) = 280$

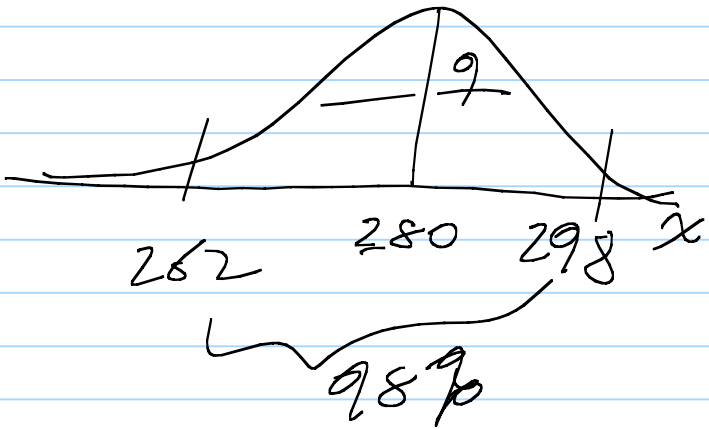
$\text{Var } X = npq = 400(.7)(.3) = 84$

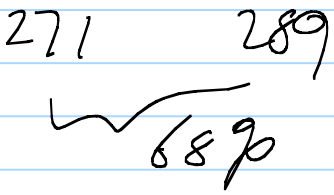
$SD\ X = \sigma_X = \sqrt{\text{Var } X} = \sqrt{84} \sim 9$

68% INTERVAL



95% INTERVAL





3. AGGREGATES OF INDEP RANDOM VARIABLES

HAVE LOTTERY RETURNS X $\boxed{EX = 15}$ $SDX = \sigma_X = 8$

ONE PLAY

$$TOTAL = X_1 + \cdots + X_{1000} \quad \text{INDEP}$$

$$E(TOTAL) = 1000(15) = 15000$$

$$\text{Var}(TOTAL) \overset{\text{INDEP}}{=} 1000(8^2)$$

$$SD(TOTAL) \overset{\text{INDEP}}{=} \sqrt{\text{Var}\,TOTAL} = \sqrt{1000\,8^2} = 8\,10\sqrt{10}$$

$$\approx$$